1.5 Ëæ×Å½üÐ©Äê´óÊý¾ÝÊ±´úµÄáÈÆð£¬Êý¾ÝµÄ·ÖÎöºÍ´¦ÀíÔÚÉç»á¿ÆÑ§¡¢ÐÅÏ¢¿ÆÑ§¡¢ÒÅ´«Ñ§¡¢ÉúÎïÑ§¡¢Ò½Ñ§ºÍ½ðÈÚÑ§µÈ¸÷´ó¿ÆÑ§ÁìÓò¶¼ÈÕÒæÊÜµ½ÖØÊÓ£¬Òò´ËÈçºÎ´Óº£Á¿Êý¾ÝÖÐ¶ÔÆä±¾ÖÊÌØÕ÷½øÐÐÌáÈ¡³ÉÎªÒ»¸öÖØÒªµÄÑÐ¾¿·½Ïò¡£²¢ÇÒ¾Íº£Á¿Êý¾Ý¶øÑÔ£¬ÎÒÃÇÈçºÎ½¨Á¢ºÏÊÊµÄÊý¾ÝÄ£ÐÍ²¢ÍÚ¾ò³öÌØÕ÷ÊýÄ¿½ÏÉÙµ«ÐÅÏ¢Ïà¶ÔÈ«ÃæµÄÊý¾ÝÀ´½øÐÐ·ÖÎö¾Í³ÉÎªÃ¿¸öÊý¾Ý¿ÆÑ§¼ÒÐèÒªÃæ¶ÔµÄÎÊÌâ¡£

1.5 ÔÚÖÚ¶àÊý¾ÝÄ£ÐÍÖÐ£¬LassoÄ£ÐÍÊÇÒ»ÖÖ¿ÉÒÔÓÐÐ§´¦Àí¸ßÎ¬Êý¾Ý£¬ÇÒ²»ËðÊ§ÏàÓ¦¾«×¼¶ÈµÄÄ£ÐÍ¡£ËüÊÇÒ»ÖÖµäÐÍµÄ±äÁ¿Ñ¡Ôñ·½·¨£¬¼´¿ÉÒÔÍ¨¹ýÉèÖÃãÐÖµ£¬ÏÞÖÆ²ÎÊý×ÜºÍµÄ´óÐ¡À´Ñ¹Ëõ²¿·Ö½ÏÐ¡±äÁ¿Îª0²¢ÌÞ³ý¶àÓàµÄ±äÁ¿¡£¶ÔÓÚ´«Í³µÄ»Ø¹éÄ£ÐÍ£¬Lasso»Ø¹éÄ£ÐÍ¼°Æä¸Ä½øµÄÄ£ÐÍÄÜ¹»ºÜºÃµÄ½â¾öÆäÔÚ±äÁ¿Ñ¡Ôñ·½ÃæµÄÎÊÌâ£¬Òò´ËLasso·½·¨¼°Æä¸Ä½øµÄ·½·¨ÔÚÍ³¼ÆÑ§ÑÐ¾¿ÖÐÊÜµ½ºÜ´óµÄÖØÊÓ¡£

1.5 ±¾ÎÄÕë¶ÔLasso»Ø¹éÄ£ÐÍ£¬Ìá³öÁËÒ»ÖÖÐÂµÄLasso¸Ä½ø·½·¨£¬¼´½«ÏÈÑéÐÅÏ¢ÈÚÈëLasso»Ø¹éÄ£ÐÍ¡£±¾ÎÄ½«Æä³ÆÎª»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso»Ø¹éÄ£ÐÍ¡£Ê×ÏÈ£¬±¾ÎÄ½éÉÜÁËLasso»Ø¹éÄ£ÐÍÔÚ»Ø¹éÎÊÌâÉÏÏà½ÏÓÚÆäËûÄ£ÐÍµÄÓÅÊÆ£¬²¢ÉîÈëÇ³³öµÄ²ûÊöÁËLasso»Ø¹éÄ£ÐÍµÄÇó½âËã·¨¼°Æä¾ßÓÐµÄÁ¼ºÃÐÔÖÊ¡£Æä´Î£¬±¾ÎÄÒýÈëÏ¡Êè¿ò¼ÜµÄ¸ÅÄîÓëLasso »Ø¹éÄ£ÐÍ½øÐÐ½áºÏ£¬½éÉÜÁËÒ»ÖÖ¸üÎªÒ»°ãµÄLasso»Ø¹é¿ò¼Ü¡£Æä¿ÉÒÔ×ª»¯ÎªÒÑÖªµÄ¶àÖÖLasso±äÐÎ¡£ÔÙ´Î£¬ÏÈÑéÐÅÏ¢×÷Îª»Ø¹é·ÖÎöÌØÕ÷×ÔÓÉÊôÐÔµÄÒ»²¿·Ö£¬Æä±¾Éí²»ÄÜÓÃÓÚÃèÊöÑÐ¾¿¶ÔÏó£¬ËùÒÔÐèÒªÍ¨¹ýÌØÓÐµÄ·½Ê½½«ÌØÕ÷±¾ÉíµÄÏÈÑéÐÅÏ¢Ó¦ÓÃµ½Ä£ÐÍÖÐÈ¥¡£Òò´Ë±¾ÎÄÒýÓÃÒ»°ãµÄLasso»Ø¹é¿ò¼Ü£¬¸ø³öÁË»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso»Ø¹éÄ£ÐÍµÄ¶¨Òå£¬²¢ÇÒ´ÓÀíÂÛÉÏ¸ø³öÁËÏàÓ¦µÄÇó½âËã·¨ºÍÐÔÖÊ¡£×îºó£¬±¾ÎÄÍ¨¹ý¶Ô¶à×éÄ£ÄâÊý¾ÝºÍÊµÖ¤Êý¾Ý½øÐÐ·ÖÎö£¬·ÖÎö½á¹û±íÃ÷ÔÚ¾ßÓÐÏÈÑéÐÅÏ¢µÄÇé¿öÏÂ£¬»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éÄ£ÐÍÏà½ÏÓÚÆÕÍ¨Lasso »Ø¹éÄ£ÐÍ¾ßÓÐ¸üºÃµÄÐÔÄÜ¡£

1.5 With the rise of the era of big data in recent years, the analysis and processing of data has received increasing attention in various scientific fields such as social sciences, information science, genetics, biology, medicine and finance, so how to deal with massive data The extraction of its essential features has become an important research direction. And in terms of massive data, how to build a suitable data model and mine data with a small number of features but relatively comprehensive information for analysis is a problem that every data scientist needs to face.

1.5 Among the many data models, the Lasso model is a model that can effectively process high latitude data without losing the corresponding accuracy. It is a typical variable selection method, which can compress some small variables to 0 and eliminate redundant variables by setting the threshold and limiting the size of the parameter sum. For the traditional regression model, the Lasso regression model and its improved model can solve the problem of variable selection well. Therefore, the Lasso method and its improved method have received great attention in statistical research.

1.5 In this paper, a new Lasso improvement method is proposed for the Lasso regression model. The prior information is incorporated into the Lasso regression model. This paper refers to the Lasso regression model based on the prior sparse framework. First of all, this paper introduces the advantages of the Lasso regression model on the regression problem compared with other models, and explains the algorithm of the Lasso regression model and its good properties. Secondly, this paper introduces the concept of sparse framework and Lasso regression model, and introduces a more general Lasso regression framework. It can be converted into a variety of known Lasso variants. Again, the prior information is part of the free attribute of the regression analysis feature, which itself cannot be used to describe the research object, so it is necessary to apply the prior information of the first feature itself to the model in a unique way. Therefore, this paper cites the general Lasso regression framework, and gives the definition of Lasso regression model based on prior sparse framework, and theoretically gives the corresponding algorithm and properties. Finally, this paper analyzes multiple sets of simulation data and empirical data. The results show that the Lasso regression model based on prior sparse framework has better performance than the ordinary Lasso regression model with prior information.

1.5 [0pt]

# Ð÷ÂÛ

## Ñ¡ÌâµÄ±³¾°¼°ÑÐ¾¿ÒâÒå

### Ñ¡Ìâ±³¾°

Ëæ×Å¿Æ¼¼µÄ½ø²½£¬ÊÕ¼¯Êý¾ÝµÄ¼¼ÊõÒ²ÓÐÁËÑ¸ÃÍµÄ·¢Õ¹£¬Òò´ËÈçºÎÓÐ¸ßÐ§¡¢¿ìËÙµØ´ÓÊý¾ÝÖÐÍÚ¾ò³öÓÐ¼ÛÖµµÄÐÅÏ¢Ô½À´Ô½ÊÜµ½ÈËÃÇµÄ¹Ø×¢¡£ÔÚ´óÊý¾ÝÊ±´ú£¬Í¨³£²ÉÓÃÍ³¼ÆÑ§ºÍ»úÆ÷Ñ§Ï°µÄ·½·¨À´¶ÔÊý¾Ý½øÐÐ´¦Àí¡£ËüÔÚºÜ¶àÁìÓòÖÐ¾ù¾ßÓÐ·Ç³£ÏÔÖøµÄÓ°Ïì£¬×î³£¼ûµÄÓÐÈË¹¤ÖÇÄÜ¡¢»úÆ÷Ê¶±ð¡¢ÒÅ´«Ñ§¡¢Ò½Ñ§¡¢½ðÈÚºÍÊÐ³¡µÈ¡£ËäÈ»Ó¦ÓÃ²ãÃæ¾ßÓÐ²»Í¬µÄÁìÓò£¬µ«ÊÇ´óÊý¾ÝÎÊÌâÈ´ÓÖ¾ßÓÐ¼¸¸ö¹²Í¬µÄÌØµã£º(1) Êý¾ÝÁ¿·Ç³£´ó£¬°üº¬¼¸°ÙÍòÉõÖÁ´ïµ½ÒÚ¼¶±ðµÄÑµÁ·Á¿£»(2) Êý¾ÝµÄÎ¬¶ÈºÜ¸ß£¬Í¨³£Ã¿¸öÑù±¾¾ù¾ßÓÐÏêÏ¸ÐÅÏ¢À´¼ÇÂ¼ÆäÌØÕ÷£»(3) ´ó¹æÄ£µÄÊý¾ÝÍ¨³£ÒÔ·Ö²¼Ê½µÄ·½Ê½´¢´æ»òÊÕ¼¯¡£ÔÚ»úÆ÷Ñ§Ï°µÄËã·¨ÖÐ£¬¶Ô×îÐÂËã·¨µÄÒªÇó¼ÈÄÜ¹»½â¾öÊý¾ÝµÄ¸´ÔÓÐÔÓÖÄÜ¹»²ÉÓÃÆ½ÐÐ»ò·Ö²¼Ê½µÄ·½·¨´¦Àí´óÊý¾Ý¡£ ¿¼ÂÇÍ¨³£µÄÏßÐÔ»Ø¹éÇéÐÎ£¬ÎÒÃÇÓÐÊý¾Ý¼¯ µÄ´óÐ¡Îª£¬×÷ÎªÏìÓ¦±äÁ¿£¬¼Ù¶¨, ÆäÖÐÔëÉùÊÇ¸ßË¹·Ö²¼£¬·ûºÏ¡£ Í¨³£µÄ×îÐ¡¶þ³Ë¹À¼ÆÊÇÍ¨¹ý×îÐ¡»¯²Ð²îÆ½·½ºÍ¶øµÄµÃµ½µÄ¡£ÏÂÃæÊÇÇó½âÏßÐÔ»Ø¹éµÄÄ¿±êº¯Êý

ËäÈ»Õâ¸ö×îÐ¡¶þ³Ë¹À¼ÆÓÐÐí¶àºÃµÄÐÔÖÊ£¬µ«ÈÔÈ»²»ÄÜÂú×ãÐí¶àÇé¿öÏÂÊý¾Ý·ÖÎöµÄÒªÇó£¬Ö÷ÒªÊÇ´æÔÚÁ½¸öÎÊÌâ¡£Ê×ÏÈÊÇÔ¤²â¾«¶ÈµÄÎÊÌâ¡£×îÐ¡¶þ³Ë¹À¼ÆËäÈ»ÊÇÎÞÆ«¹À¼Æ£¬µ«ÊÇËüµÄ·½²îÔÚ×Ô±äÁ¿ÏßÐÔÏà¹Ø³Ì¶È¸ßÊ±Í¨³£½Ï´ó£¬¿ÉÒÔÍ¨¹ý½«Ä³Ð©Ï¡ÊèÑ¹Ëõµ½0À´¸Ä½øÕâ¸öÔ¤²â¾«¶È¡£Æä´Î£¬ÊÇÄ£ÐÍµÄ¿É½âÊÍÐÔ£¬¶ÔÓÚ×Ô±äÁ¿¸öÊýºÜ¶àµÄÇé¿öÏÂ£¬ÎÒÃÇ×ÜÊÇÏ£ÍûÈ·¶¨Ò»¸ö½ÏÐ¡µÄ±äÁ¿Ä£ÐÍÀ´±íÏÖ³ö×îºÃµÄÐ§¹û¡£ ÓÐÁ½ÖÖ·½·¨¿ÉÒÔ¶Ô×îÐ¡¶þ³Ë¹À¼Æ½øÐÐ¸Ä½ø£¬·Ö±ðÊÇ×Ó¼¯Ñ¡ÔñºÍÁë»Ø¹é£¬µ«ÊÇËûÃÇ¶¼ÓÐ¸÷×ÔµÄÈ±µã¡£×Ó¼¯Ñ¡ÔñËäÈ»Ê¹Ä£ÐÍ¿É½âÊÍ£¬µ«È´Ê¹Ä£ÐÍ±äµÃ²»ÎÈ¶¨£¬ÕâÊÇËüµÄÀëÉ¢ÐÍ³ÌÐò¾ö¶¨µÄ¡ª¡ª »Ø¹éÏµÊýÒªÃ´ÊÇ±»±£ÁôÒªÃ´¾ÍÊÇ¼òµ¥µØ´ÓÄ£ÐÍÖÐÌÞ³ý£¬ÕâºÜ¿ÉÄÜÊ¹µÃ¹Û²âÊý¾ÝµÄÒ»¸öÎ¢Ð¡±ä¶¯¾Íµ¼ÖÂÒªÑ¡ÔñÒ»¸öÐÂµÄÄ£ÐÍ£¬´Ó¶øÓ°ÏìÁËÔ¤²âµÄ×¼È·ÐÔ¡£¶øÁë»Ø¹éÊÇÒ»¸öÁ¬ÐøÐÍµÄ·½·¨£¬ËüËõÐ¡ÁË»Ø¹éÏµÊý£¬¶øÇÒÃ»ÓÐ¼òµ¥µÄÅ×µôÄÇ¸ö±äÁ¿£¬Ä£ÐÍ±È½ÏÎÈ¶¨£¬µ«ÕýÊÇÓÉÓÚËüÃ»ÓÐÈÃÈÎºÎÒ»¸ö»Ø¹éÏµÊý¼õÉÙµ½0£¬Ê¹µÃÄ£ÐÍÖÐ±äÁ¿Ì«¶à£¬Ä£ÐÍµÄ½âÊÍÐÔ²»ºÃ¡£ Lasso ÊÇ1996 ÄêTibshiraniÌá³öµÄÒ»ÖÖ·½·¨£¬Lasso µÄÄ¿±êÊÇÎªÁË½â¾öÏÂÃæÕâ¸öÎÊÌâ

Lasso°üº¬Æ½·½Îó²îµÄÇóºÍÒÔ¼°L1ÕýÔò»¯Ïî¡£ÕâÖÖ·½·¨ÓÃÄ£ÐÍÏµÊýµÄ¾ø¶ÔÖµº¯Êý×÷Îª³Í·£À´Ñ¹ËõÄ£ÐÍÏµÊý£¬Ê¹Ò»Ð©»Ø¹éÏµÊý±äÐ¡£¬ÉõÖÁ»¹Ê¹Ò»Ð©¾ø¶ÔÖµ½ÏÐ¡µÄÏµÊýÑ¹³ÉÁË0.Lasso ·½·¨¼¯ºÏÁË×Ó¼¯Ñ¡Ôñ·¨ºÍÁë»Ø¹é¸÷×ÔµÄÓÅµã£¬¼È¿ÉÒÔ½«Ò»Ð©Òò×ÓµÄÏµÊýÑ¹Ëõµ½0£¬ÓÖ±£³ÖÁËÁ¬ÐøÐÔ£¬²¢ÎªÄ£ÐÍÌá¹©ÁË¸üºÃµÄ½âÊÍÐÔºÍÔ¤²âÐÔ¡£Òò´ËÄ¿Ç°¾­³£ÀûÓÃLassoÀ´´¦Àí¹²ÏßÐÔÎÊÌâºÍ±äÁ¿Ñ¡ÔñÎÊÌâ¡£

±¾ÎÄÏ¡Êè¿ò¼ÜÔÚ´ËÖ¸µÄÊÇÏ¡Êè¿ò¼Ü¾ØÕó£¬ÆäÊÇÓÃÓÚÃèÊöÏ¡Êè±àÂëÀ´±íÊ¾ÕæÊµµÄ£¬ÆäÄ¿±êº¯ÊýÎª

ÆäÖÐTÊÇÖ¸¶¨µÄÏ¡Êè¾ØÕó£¬µ±TÎªµ¥Î»¾ØÕóÊ±£¬¸ÃÄ£ÐÍ¼ÈÎªLassoÄ£ÐÍ¡£¶ÔÓÚT¾ØÕó½øÐÐ×ª»¯¿ÉÒÔ×ª»¯³É»ù±¾ËùÓÐµÄ¸Ä½øºóµÄLassoÄ£ÐÍ¡£

### ÀíÂÛÒâÒå

ÔÚÍ³¼Æ½¨Ä£ÖÐ£¬Lasso·½·¨ÄÜ¹»ºÜºÃµÄ½øÐÐ±äÁ¿Ñ¡ÔñºÍ±äÁ¿µÄÑ¹Ëõ£¬Ä¿Ç°ÎÞÂÛÊÇÔÚÑ§ÊõÑÐ¾¿ÖÐ»¹ÊÇÊµ¼ÊÓ¦ÓÃÖÐ¶¼³ÉÎªÁËÐÐÖ®ÓÐÐ§µÄ·½·¨¡£Òò´ËÍ¨¹ýÏ¡Êè¿ò¼ÜºÍLasso ·½·¨ºÜ×ÔÈ»µÄ½øÐÐ½áºÏ£¬ÀûÓÃÒÑÖªµÄÏÈÑéÐÅÏ¢¶ÔLasso»Ø¹é½øÐÐ¸Ä½ø£¬Ê¹ÆäÔ¤²âµÄ½á¹û¸ü¼Ó¾«×¼£¬²»Ê§Îª¶ÔÔ­Ê¼·½·¨½øÐÐÒ»ÖÖ´´ÐÂ£¬Ò²ÄÜ¹»ÔÚÔ­À´µÄË¼ÏëÉÏ´øÈëÒ»Ð©ÐÂµÄË¼Â·ÓëÏë·¨¡£

### ÏÖÊµÒâÒå

ÎÒÃÇ²»½ö¿ÉÒÔÀûÓÃÏ¡Êè¿ò¼Ü¶ÔLassoµÄ¹À¼Æ·½·¨½øÐÐÒ»ÏµÁÐµÄ´¦ÀíµÃµ½¸Ä½ø£¬Í¬Ê±°éËæ×Å»úÆ÷Ñ§Ï°µÄ²»¶Ï·¢Õ¹£¬Éî¶ÈÑ§Ï°µÄÖð½¥ÐËÆð£¬ÒÔ¼°ÁÄÌì»úÆ÷ÈË¡¢Alpha Go¡¢ÎÞÈË¼ÝÊ»¡¢ÎÞÈË»úµÈÒ»ÏµÁÐÈË¹¤ÖÇÄÜ²úÎïµÄµ®Éú£¬ÈËÃÇµÄÊÓÏßÒ²Öð½¥±»ÎüÒý¹ýÀ´¡£ÎÒÃÇ¿ÉÒÔÀûÓÃÏ¡Êè¿ò¼Ü×ÔÉíËù¾ß±¸µÄÊµÓÃÐÔºÍ¿ÉËÜÐÔ£¬²»½öÔÚÀíÂÛÉÏ¿ÉÒÔÊµÏÖÒ»¶¨µÄ½ø²½£¬Í¬Ê±Ò²ÄÜ¹»Óëµ±Ç°µÄ¼¼ÊõÏà½áºÏ£¬Òò´ËÒ²¾ß±¸Ò»¶¨µÄÊµ¼ÊÒâÒå¡£ ÓÚ´ËÍ¬Ê±£¬Lasso×÷ÎªÒ»ÖÖ¸Ä½øµÄ×îÐ¡¶þ³Ë¹À¼Æ£¬ÊÇ×î»ù±¾¡¢×î³£ÓÃµÄÌØÕ÷Ñ¡ÔñµÄ·½·¨£¬¾ßÓÐÁ¬ÐøËõÐ¡¡¢·º»¯Îó²îµÈÓÅµã£¬ÔÚ½¨Ä£ºÍÔ¤²âÑ§Ï°ÖÐ¾ßÓÐºÜÇ¿µÄÎüÒýÁ¦£¬ÔÚÊµ¼ÊÓ¦ÓÃÖÐ·Ç³£ÓÐÐ§¡£ÓÈÆäÊÇ¶ÔÓÚÒ»¸öÐ¡n£¬´ópµÄÎ¢¾ØÕóÊ±£¬Lasso·½·¨ÄÜ¹»ºÜºÃµØÉ¸Ñ¡³öÖØÒªµÄÌØÕ÷£¬²¢ÇÒÈç¹ûÎÒÃÇ¶ÔÓÚ¹À¼Æ²ÎÊý¾ßÓÐÒ»¶¨µÄÏÈÑéÐÅÏ¢£¬ÓëLasso·½·¨Ïà½áºÏ£¬ÎÒÃÇ¼È¿ÉÒÔÉ¸Ñ¡²»ÒªµÄÌØÕ÷£¬ÓÖ¿ÉÒÔÊ¹ÎÒÃÇÐèÒªµÄÌØÕ÷Ô¤²âµÄ¸ü¼Ó×¼È·£¬ÄÜ¶ÔÎÒÃÇ×îÖÕµÄÉ¸Ñ¡½á¹û¾ßÓÐ¸ü¸ßµÄÒ»¸öÔ¤²â¾«¶È£¬¸üÐ¡µÄ·º»¯Îó²î¡£×ÛºÏÉÏËß·ÖÎöLasso »Ø¹éÄ£ÐÍÔÚÊµ¼ÊÓ¦ÓÃÖÐÓÐ×ÅÊ®·Ö¾Þ´óµÄÒâÒå£¬Òò´Ë£¬½áºÏÏ¡Êè¿ò¼ÜºÍÏÈÑéÐÅÏ¢µÄLasso Í¬Ñù¾ßÓÐÏàÓ¦µÄÊµ¼ÊÓ¦ÓÃ¡£

## ¹úÄÚÍâÎÄÏ××ÛÊö

LassoÄ£ÐÍÊ¹ÓÃÁËL1·¶Êý×÷Îª³Í·£Ïî£¬ÕâÖÖ³Í·£ÏîÒÑ¹ã·ºÓ¦ÓÃÓÚÍ³¼ÆÑ§¡¢»úÆ÷Ñ§Ï°¡¢½ðÈÚµÈ¶à¸öÁìÓò¡£LassoÄ£ÐÍÊÇ1996ÄêÓÉTibshirani(1996)Ìá³öÀ´µÄ£¬ËûÊÇÊ¹ÓÃL1³Í·£Òò×Ó½â¾öº¬ÓÐ¶ÀÁ¢¸ßË¹·Ö²¼µÄÏßÐÔ»Ø¹éÎÊÌâ£¬ËäÈ»LassoÄ£ÐÍ³öÏÖµÄ±È½ÏÍí¡£µ«ÊÇ¶ÔÓÚLasso Ä£ÐÍµÄÑÐ¾¿ºÍÌ½Ë÷Ò»Ö±ÔÚ½øÐÐ×Å¸ßËÙµÄ·¢Õ¹¡£Ò²³öÏÖÁËºÜ¶à¸Ä½øºóµÄÄ£ÐÍ£¬Ëã·¨£¬²¢ÇÒÓ¦ÓÃÓÚ¶à¸öÁìÓò¡£

µ¯ÐÔÍøÄ£ÐÍÊÇZou(2005)Ìá³öÀ´µÄÒ»ÖÖLassoµÄ±äÌå£¬ÊÇÁë»Ø¹éºÍLasso»Ø¹éµÄÒ»ÖÖÕÛÖÐÄ£ÐÍ£¬ËüµÄ³öÏÖÊÇÎªÁË½â¾öLasso²»ÉÃ³¤´¦ÀíµÄ¸ß¶ÈÏà¹Ø±äÁ¿¡£ÈÚºÏLasso ÊÇÓÉTibshirani(2005) Ìá³öÀ´µÄ£¬Æä×îÔçÊÇÓÃÓÚ½«»ùÒòµÄÈ¾É«Ìå½øÐÐ½µÔë´¦Àí¶ø²úÉúµÄ,ÏÖÔÚ²»Í¬µÄ°æ±¾µÄÈÚºÏLassoÒÑ¾­ÓÐ¶àÖÖÇó½âËã·¨(Tibshirani µÈ,2005,HoeflingµÈ,2010,TibshiraniµÈ,2011)¡£ Yuan(2006a,2006b,2006c) ÒýÈëÁË×éLasso£¬ÎªÁË½â¾öÐ­±äÁ¿±¾Éí¾ßÓÐ×é½á¹¹£¬ËûÃÇ·¢±íµÄÂÛÎÄÔÚµ±Ê±ÒýÆðÁËÑÐ¾¿ÈÈ³±¡£She(2010)Ìá³öÁË¾ÛÀàLasso£¬ÆäÔÚÈÚºÏLasso µÄ»ù´¡ÉÏ¸ü¼Ó½øÒ»²½µÄ¶ÔÌØÕ÷½øÐÐ´¦Àí£¬ÊÔÍ¼Ê¹·ÇÁãÏµÊýÔÚ¾ÛÀàÖÐÏàµÈ£¬²¢ÇÒ±£³ÖÄ£ÐÍµÄÏ¡ÊèÐÔ¡£ÆäÒýÓÃÁËÏ¡Êè¿ò¼ÜÎªÁË¸üºÃµÄ±íÊ¾¾ÛÀàLasso¡£Negahban(2012) Ìá³öÁËÒ»¸öM ¹À¼Æ·ÖÎöµÄÒ»°ãÐÔ¿ò¼Ü£¬ÆäÖÐ×éLasso Ö»ÊÇÒ»¸öÌØÀý¡£µ«ÊÇÉÏÊöµÄÄ£ÐÍ¶¼Ã»ÓÐ¿¼ÂÇ¹ýÄÜ·ñÀûÓÃ²ÎÊý×ÔÉí¾ß±¸µÄÄ³Ð©ÏÈÑéÐÅÏ¢£¬½«Æä¸üºÃµÄ·¢¾òºÍÀûÓÃ£¬Òò´Ë£¬±¾ÎÄÖØµãÑÐ¾¿µÄ¾ÍÊÇ»ùÓÚLassoÄ£ÐÍ¾ßÓÐÏÈÑéÐÅÏ¢µÄÇé¿öÏÂ½øÐÐµÄÒ»ÖÖ¸Ä½øºÍ´´ÐÂ¡£

²¢ÇÒLassoÄ£ÐÍÔÚ¶à¸öÁìÓòÖÐÆð×Å·Ç³£ÖØÒªµÄ×÷ÓÃ¡£ÔÚ½â¾öÏ¡Êè¹æ»®ÎÊÌâÊ±£¬Ëû¿ÉÒÔ½âÊÍÎªÑ°ÕÒ×îÐ¡¶þ³Ë»òÏßÐÔ»Ø¹éÎÊÌâµÄÏ¡ÊèÎÊÌâ£¬¼´²ÉÓÃÈô¸É¸ö·ÇÁãµÄ±äÁ¿¡£LassoÎÊÌâÔÚÐÅºÅ´¦ÀíÁìÓòÖÐÍ¬ÑùÆð×ÅÖØÒªµÄ×÷ÓÃ£¬°üÀ¨½â¾öÏ¡ÊèÄæÐ­·½²î(Yuan µÈ,2012)¡¢Ï¡ÊèÍ¼»Ø¹é(Zhou µÈ£¬2014) ºÍÏ¡Êè×ÖµäµÄÑ§Ï°(SunµÈ,2014)¡£ Tibshirani(2005) ½«LassoÓ¦ÓÃÓÚÉúÎïÊý¾ÝµÄ·ÖÎö, ±ÈÈçÑ¡È¡´óÊý¾ÝÖÐµÄÒ»Ð¡²¿·ÖÀ´Ô¤²â½á¹û¡£ÓÚ´ËÍ¬Ê±£¬LassoÄ£ÐÍÍ¬ÑùÓ¦ÓÃÓëÊÓÆµ·ÖÎöÖÐ£¬Geng(2012)²ÉÓÃ²¢ÐÐLasso µÄ·½·¨À´½â¾ö´ó¹æÄ£ÊÓÆµ¸ÅÄî¼ì²âZhu(2013)Ê¹ÓÃGroup Lasso ½â¾öÊÓÆµ±êÇ©ÒÔ¼°Zhou(2013) ÓÃÏ¡ÊèÀëÈºÖµ·Ö¸îÒÆ¶¯Ä¿±ê¡£ÔÚÍ¼Ïñ´¦ÀíÖÐ£¬Afonso(2010)Ê¹ÓÃLasso ·½·¨½â¾öÍ¼ÏñÐÞ¸´ÎÊÌâ£¬Í¼ÏñÈ¥Ôë(Elad µÈ,2006) ºÍÈ¥Ä£ºý¿Ú(FigueiredoµÈ,2003)£¬ÆäÖÐÕýÔò»¯ÏîÊÇÍ¼ÏñµÄÌÝ¶È·¶Êý¡¢ÍøÒ³Í¼ÏñÅÅÐò(Yu µÈ,2014)¼°Í¼ÏñÖÊÁ¿ÆÀ¹À(He µÈ,2012)£¬ÀûÓÃÏ¡Êè±àÂë´¦ÀíÍ¼Ïñ±êÇ©ÎÊÌâ(Dupe µÈ£¬2009)¡£ ÔÚÒ£¸ÐÁìÓòÖÐ£¬Lasso ÎÊÌâÓÃÀ´½â¾öÏ¡Êè½â(Bioucas-DiasµÈ,2010,Plaza A,2011,Richards,1999)£¬´Ó¶ø¿ÉÒÔµÃµ½Ã¿¸ö¶ËÔª¶¼º¬ÓÐÄÄÐ©ÎïÖÊ£¬Í¬Ê±ÔÚº½Ìì¡¢Å©ÒµÁìÓò£¬¶ÔÑÐ¾¿ÎïÖÊ³É·ÖÎÊÌâÆð×Å·Ç³£ÖØÒªµÄ×÷ÓÃ¡£

½â¾öLassoÄ£ÐÍµÄ·½·¨Ò²ÓÐºÜ¶à£¬ÆäÖÐTibshiraniÔÚÔ­ÂÛÎÄÖÐÌá³öÒ»ÖÖ¿ÉÒÔ½â¾öLasso ÎÊÌâµÄËã·¨£¬µ«ÊÇ¸ÃËã·¨Çó½âËÙ¶ÈÌ«Âý¡£½ô½Ó×Å£¬Efron(2004) Ìá³öÁË×îÐ¡½Ç»Ø¹é£¬ËüÄÜ¹»ÓÐÐ§µØ½â¾öÇ±ÔÚµÄLasso ÓÅ»¯ÎÊÌâ£¬Í¬Ê±ÔÚÍ³¼ÆºÍ»úÆ÷Ñ§Ï°ÁìÓò£¬ËüÎª¼ÓËÙLasso Ëã·¨Ìá¹©ÁËÒ»¸öÖØÒªµÄ¹¤¾ß¡£ÔÚ½â¾ö´ó¹æÄ£Êý¾ÝÎÊÌâÖÐ£¬ÌÝ¶ÈÏÂ½µ·½·¨ÊÇ×î¼òµ¥µÄ·½·¨£¬Nesterov(2004,2005) Ìá³öÁË×îÓÅµÄÌÝ¶ÈÏÂ½µ·½·¨ºÍ¹â»¬Ä£ÐÍ£¬ÆäÔÚLasso Ä£ÐÍÖÐ±»¹ã·ºµÄÓ¦ÓÃ¡£ÓÚ´ËÍ¬Ê±£¬ÓÉÓÚL1 ³Í·£ÏîµÄ²»¿Éµ¼£¬ÀûÓÃ×ø±êÏÂ½µ·¨¿ÉÒÔºÜºÃµÄ½â¾öÕâ¸öÎÊÌâ£¬×ø±êÏÂ½µ·¨×îÔçÊÇFriedman(2007)·¢Ã÷µÄ£¬¶øWu(2008)Ôò½«ÆäÔËÓÃµ½Lasso Ä£ÐÍµ±ÖÐ£¬Nesterov(2012) ÀûÓÃ×ø±êÏÂ½µ·¨¸Ä½øÁËÌÝ¶ÈÏÂ½µ·¨µÄ²»×ã¡£Ê¹µÃÆäÔÚ´¦ÀíLasso ÎÊÌâÊ±£¬ÄÜ¹»¸üºÃµÄÊÊÓ¦´ó¹æÄ£µÄÊý¾Ý£¬ÆäÖÐSaha(2013) ¶ÔÓÚ×ø±êÏÂ½µ·¨µÄÊÕÁ²ÐÔÖÊÒÑ¾­¸ø³öÁËÏêÏ¸µÄ·ÖÎö¡£½üÐ©Äê£¬Boyd(2010) Ìá³ö½»Ìæ·½Ïò³Ë×Ó(Alternating direction method of multipliers£¬ADMM)·½·¨, ½áºÏ·Ö²¼Ê½Í¹ÓÅ»¯ÎÊÌâ£¬Í¬ÑùÊÊºÏ½â¾ö´ó¹æÄ£Êý¾ÝµÄÎÊÌâ£¬²¢ÇÒÒÑ¾­³É¹¦µØÓ¦ÓÃÔÚºÜ¶àÁìÓò£¬°üÀ¨½â¾öLassoÄ£ÐÍµÄÇó½âÎÊÌâ¡£ÓÚ´ËÍ¬Ê±£¬Goldstein T(2014) Ìá³öÁË¿ìËÙADMM·½·¨¡£×î½üµÄÑÐ¾¿ÕßSuzuki(2014)½áºÏ´óÊý¾ÝµÄÌØµã²ÉÓÃËæ»úµÄ·½·¨£¬ÔÚÔ­ÓÐËã·¨µÄ»ù´¡ÉÏ£¬Ìá¸ßÁËËã·¨µÄÊÕÁ²Ð§ÂÊ¡£

Í¨¹ýÊáÀíÉÏËßÇó½âLassoÄ£ÐÍµÄ·½·¨ºó£¬±¾ÎÄÐèÒªÇóÖ¤µ±Ç°µÄÖ÷Á÷Çó½â·½·¨ÊÇ·ñÊÊÓÃÓÚ±¾ÂÛÎÄ¸Ä½øºóLassoÄ£ÐÍ¡£Èç¹û²»ÊÊÓÃÐèÒªÔÚÔ­ÓÐµÄÇó½â·½·¨»ù´¡ÉÏ½øÐÐ¸Ä½ø£¬²¢ÇÒ¶Ô¸÷¸ö·½·¨µÄÊÕÁ²Ê±¼äºÍ×¼È·¶È½øÐÐÒ»¶¨µÄ¶Ô±È¡£

## ±¾ÎÄ½á¹¹

µÚ1ÕÂÊÇÐ÷ÂÛ²¿·Ö£¬ÔÚ±¾ÕÂµÄ¿ªÊ¼½éÉÜÁËLasso¹À¼ÆÎÊÌâºÍÔÚÏ¡Êè¿ò¼ÜÏÂ¾ßÓÐÏÈÑéÐÅÏ¢·½·¨µÄÑÐ¾¿±³¾°¼°ÑÐ¾¿ÒâÒå£¬ÔÚÑÐ¾¿ÒâÒåÖÐ°üº¬ÁËÀíÂÛÒâÒåºÍÏÖÊµÒâÒå¡£½ÓÏÂÀ´½éÉÜÁË¹úÄÚÍâÔÚ´Ë·½ÃæµÄÑÐ¾¿×´¿ö¡£Ëæºó¶ÔÏà¹ØµÄ±³¾°ÖªÊ¶×öÁË¼òµ¥µÄ½éÉÜ£¬°üÀ¨×îÐ¡¶þ³Ë¹À¼Æ·½·¨ºÍÁë»Ø¹é¡£

µÚ2ÕÂÈ«Ãæ²ûÊöÁËLasso»Ø¹éÄ£ÐÍ£¬Ê×ÏÈ¶ÔÓÚLasso»Ø¹éÄ£ÐÍµÄ±³¾°×öÁË½éÉÜ£¬½ô½Ó×Å¸ø³öÁËLasso»Ø¹éÄ£ÐÍµÄ¶¨Òå£¬²¢ÇÒ¼òÒªÖ¤Ã÷ÁËÁ½ÖÖ²»Í¬ÐÎÊ½ÏÂµÄLasso»Ø¹éÄ£ÐÍ¾ßÓÐÒ»Ò»¶ÔÓ¦µÄ¹ØÏµ¡£ÔÚÕâÖ®ºó¸ø³öÁËLasso»Ø¹éµÄÈýÖÖÇó½â·½·¨£ºLassoËã·¨¡¢×îÐ¡½Ç»Ø¹éËã·¨ºÍ×ø±êÏÂ½µ·¨£¬Óë´ËÍ¬Ê±¸ø³öÁËÃ¿ÖÖËã·¨µÄ¾ßÌå²½ÖèºÍÓÅÈ±µã¡£×îºó£¬±¾½ÚÕë¶ÔLasso »Ø¹éÄ£ÐÍµÄÐÔÖÊ½øÐÐÁË¹éÄÉ×Ü½á£¬²¢ÇÒ¸ø³öÁËÏàÓ¦µÄÖ¤Ã÷¡£

µÚ3ÕÂÑÐ¾¿ÁË±¾ÎÄµÄÀíÂÛÑÐ¾¿²¿·Ö£¬Ê×ÏÈ±¾ÕÂÒýÈëÁËÒ»ÖÖ¸üÎª¿í·ºµÄLasso »Ø¹éÄ£ÐÍ£¬¼´»ùÓÚÏ¡Êè¿ò¼ÜµÄLasso»Ø¹é£¬²¢ÔÚ´ËÖ®ÉÏÈÚÈëÁËÏÈÑéÐÅÏ¢µÄË¼Ïë¡£½ô½Ó×Å¸ø³öÁËÐÂÄ£ÐÍµÄ¾ßÌåÐÎÊ½£¬²¢ÇÒ¾ÍÁ½ÖÖ²»Í¬ÐÎÊ½µÄ¸ø³öÁËÏàÓ¦µÄÇó½âËã·¨£¬×îºóËµÃ÷ÁËÐÂÄ£ÐÍËù¾ßÓÐµÄÁ¼ºÃÐÔÖÊ£¬ÎªºóÎÄµÄÊýÖµÊµÑé×ö³öÀíÂÛÖ§³Å¡£

µÚ4ÕÂÊÇ¶ÔÐÂÄ£ÐÍ½øÐÐÊý¾ÝÊµÑé£¬±¾ÕÂ½«´«Í³Lasso Ä£ÐÍºÍÐÂÄ£ÐÍÔÚÄ£ÄâºÍÊµÀýÊý¾ÝÉÏ·Ö±ð½øÐÐÑÐ¾¿¡£¾ßÌåµØ£¬ÔÚÄ£ÄâÊý¾ÝÊµÑéÖÐÎÒÃÇ¼ÆËã²¢±È½Ï¸÷¸öÄ£ÐÍµÄ¾ù·½Îó²îºÍÔ¤²âÎó²î£¬²¢ÇÒ»­Í¼ÃèÊöÁË¸÷Ä£ÐÍ¼ÆËãµÄ²ÎÊýÖµÓëÕæÊµ²ÎÊýµÄÇ÷½ü×´¿ö¡£ÔÚÊµÀýÊý¾Ý·ÖÎöÖÐÎÒÃÇ¼ÆËã²»Í¬Ä£ÐÍµÄÆ½¾ùÔ¤²âÎó²îºÍÆ½¾ù¾ø¶ÔÎó²î£¬±íÏÖÆäÔÚÕæÊµÊý¾ÝÖÐµÄ¿ÉÐÐÐÔ¡£

µÚ5ÕÂÊÇ×Ü½áºÍÕ¹Íû£¬×Ü½á±¾ÎÄËùÑÐ¾¿µÄÖ÷ÒªÎÊÌâ¡¢·½·¨Óë½á¹û£¬²¢ÃèÊöÎ´À´µÄÀ©Õ¹ÐÔ¹¤×÷ÓëÑÐ¾¿·½Ïò¡£

## ¿ÎÌâÏà¹Ø±³¾°ÖªÊ¶½éÉÜ

### ×îÐ¡¶þ³Ë¹À¼Æ

ÔÚÏßÐÔ»Ø¹éÄ£ÐÍÖÐµÄ»Ø¹éÏµÊýµÄ¹À¼ÆÎÊÌâÒ»Ö±ÊÇÖÚ¶àÑ§ÕßÑÐ¾¿·ÖÎöµÄ¿ÎÌâ£¬ÆäÖÐ×îÔ­Ê¼Ò²ÊÇ×î»ù±¾µÄ¹À¼Æ·½·¨¾ÍÊÇ×îÐ¡¶þ³Ë¹À¼Æ£¬ËüÍ¨¹ý×îÐ¡»¯Îó²îµÄÆ½·½ºÍÑ°ÕÒÊý¾ÝµÄ×î¼Ñº¯ÊýÆ¥Åä£¬ÒÑ´ïµ½×îÖÕ½â¾öÎÊÌâµÄÄ¿µÄ¡£

Ê×ÏÈ¸ø³öÒ»¸öÏßÐÔÄ£ÐÍ¡£ÆäÖÐ´æÔÚ×Åm ¸ö×Ô±äÁ¿ ºÍÒò±äÁ¿, ¾ßÓÐÈçÏÂµÄÏßÐÔ¹ØÏµ

ÆäÖÐÎªÎ´Öª²ÎÊý£¬Ò²ÊÇÏßÐÔÄ£ÐÍÖÐÒª¹À¼ÆµÄ¶ÔÏó£¬ÊÇÆÚÍûÎª0 ·½²îÎª µÄÎó²îÏî.

Èô¶ÔÓÚ´ËÏßÐÔÄ£ÐÍÓÐn×é¹Û²âÖµ£¬¼´,i=0,1,,n, ÄÇÃ´Îó²îÏî,²¢ÇÒÓÐ£¬ÔòµÚi×é¹Û²âÖµµÄÏßÐÔ¹ØÏµÎª

ÆäÖÐ¾ØÕóÐÎÊ½Îª

½«´Ë·½³Ì¼ò»¯Îª

×îÐ¡¶þ³Ë¹À¼ÆµÄÔ­ÀíÎªÊ¹µÃ¹«Ê½[[1.1]](#X491f8b5ecc09d9de308ea37e6a23ad2dd66c256)´ïµ½×îÐ¡Öµ£¬Í¨¹ýÇóÆ«µ¼µÃµ½²ÎÊý¹À¼Æ

Ê¹ÓÃ×îÐ¡¶þ³Ë¹À¼ÆºóµÄ²ÎÊý¹À¼Æ¾ßÓÐÈçÏÂÐÔÖÊ£º

**ÐÔÖÊ1.1** µÄ¹À¼ÆÊÇµÄÎÞÆ«¹À¼Æ.

**ÐÔÖÊ1.2** Èô,ÔòÓÐ.

**ÐÔÖÊ1.3** ¼ÙÉèbÎªµÄ³£ÊýÏòÁ¿£¬ÄÇÃ´¶ÔÓÚÏßÐÔº¯Êý, ³Æ ÎªµÄ×îÐ¡¶þ³Ë¹À¼Æ£¬ÄÇÃ´ÓÐ

**ÐÔÖÊ1.4** ²Ð²îÆ½·½ºÍÎªRSS£¬ÔòÓÐ ÊÇ µÄÎÞÆ«¹À¼Æ

### Áë»Ø¹é¹À¼Æ

ÉÏÒ»½ÚËµµ½×îÐ¡¶þ³Ë¹À¼ÆµÄÔ­ÀíÊÇÊ¹µÃ ´ïµ½×îÐ¡Öµ£¬ÎÒÃÇ½«Æä×÷Îª¶ÈÁ¿£¬ÒýÈë¾ù·½Îó²î(Mean Squared Error) À´ºâÁ¿²ÎÊý¹À¼ÆÁ¿½Ó½üÕæÊµ²ÎÊýµÄ³Ì¶È

Òò´Ë£¬ÎªÁËÊ¹¾ù·½Îó²î×îÐ¡»¯£¬¿ÉÒÔÊÓÎªÔÚ¹À¼ÆÁ¿·½²îÓëÆ«²îÖ®¼ä½øÐÐÈ¨ºâ¡£ÓÚ´ËÍ¬Ê±ÎÒÃÇÒýÈëÔ¤²âÎó²î(Predicted Error)µÄ¶¨Òå

ÓÉÉÏÊ½¿ÉÖª£¬ÔÚ¸ø¶¨µÄÇ°ÌáÏÂ£¬MSE×îÐ¡»¯µÈ¼ÛÓÚPE×îÐ¡»¯¡£ ÔÚ¶àÔªÏßÐÔ»Ø¹éÄ£ÐÍÖÐ£¬ÓÃ×îÐ¡¶þ³Ë¹À¼ÆµÃµ½µÄ»Ø¹éÏµÊý¹À¼ÆÁ¿ÊÇ¾ßÓÐÎÞÆ«ÐÔµÄÇÒ¾ßÓÐ×îÐ¡·½²î¡£µ«ÊÇ²¢²»ÄÜËµÃ÷×îÐ¡¶þ³Ë¹À¼ÆÁ¿µÄ·½²îÒ»¶¨ÊÇ×îÐ¡µÄ¡£ÓÈÆäÊÇÔÚÄ£ÐÍ´æÔÚ¶àÖØ¹²ÏßÐÔµÄÎÊÌâÊ±ºò£¬Ò»¸öÉÔÉÔÓÐÆ«µÄ¹À¼ÆÁ¿£¬Æä¾«¶È¿ÉÄÜÔ¶Ô¶¸ßÓÚÎÞÆ«¹À¼ÆÁ¿¡£ÓÚÊÇÌá³öÁËÒ»ÖÖÒÔ·ÅÆú¶ÔÏßÐÔ»Ø¹éÄ£ÐÍÖÐµÄ»Ø¹éÏµÊýÒ»°ã×îÐ¡¶þ³Ë¹À¼ÆµÄÎÞÆ«ÐÔÒªÇóµÄ·½·¨£¬Ò²¾ÍÊÇÁë»Ø¹é¹À¼ÆµÄµ®Éú¡£

ÔÚ¶àÔªÏßÐÔ»Ø¹éÄ£ÐÍÖÐ£¬»Ø¹é·½³Ì³öÏÖ¶àÖØ¹²ÏßÐÔÊ±£¬»á´æÔÚ µÄÇé¿ö£¬ÕâÑùµÄÇé¿ö»áÊ¹µÃ²ÎÊý¹À¼Æ

±äµÃºÜ²»ÎÈ¶¨¡£²¢ÇÒ»áµ¼ÖÂµÄ¾ù·½Îó²îÇ÷ÏòÓÚÎÞÇî´ó¡£ÕâÊ±£¬Èç¹ûÔÚ ÖÐ¼ÓÈëÒ»¸ö³£Êý¾ØÕó(),ÄÇÃ´ÎÒÃÇ¾Í¿ÉÒÔµÃµ½ ÐÎ³ÉµÄ¾ØÕóÒ»¶¨ÊÇ·ÇÆæÒìµÄ¡£

´Ó¶øµÃµ½Áë»Ø¹é¹À¼Æ(ArthurµÈ,1970)µÄ²ÎÊý±í´ïÊ½Îª

ÆäÖÐµÄ²ÎÊý±»³Æ×÷Áë²ÎÊý¡£Òò´Ë£¬ÏßÐÔ»Ø¹é·½³ÌµÄÄ¿±êº¯ÊýÔòÓÉÔ­À´µÄ¹«Ê½[[1.1]](#X491f8b5ecc09d9de308ea37e6a23ad2dd66c256) ×ª±äÎª

Áë»Ø¹é¹À¼Æ¾ßÓÐÒÔÏÂ¼¸µãÐÔÖÊ£º

**ÐÔÖÊ1.5** µ±Áë²ÎÊý±»ÈÏÎªÊÇºÍ ÎÞ¹ØµÄ²ÎÊýÊ±£¬Áë»Ø¹é¹À¼ÆµÄ ÊÇ×îÐ¡¶þ³Ë¹À¼ÆµÄÏßÐÔ±ä»»¡£

**ÐÔÖÊ1.6** ,ÆäÖÐ, Ö»Òª,Áë»Ø¹é¹À¼ÆµÄ ¾ÍÊÇµÄÓÐÆ«¹À¼Æ

**ÐÔÖÊ1.7** Áë»Ø¹é¹À¼ÆÊÇÏòÔ­µãµÄÑ¹Ëõ£¬¼´¶ÔÈÎÒâ, ÇÒ µÄÇé¿ö£¬×ÜÓÐ.

**ÐÔÖÊ1.8** ´æÔÚ£¬ÔÚ¾ù·½Îó²î×¼ÔòÏÂ£¬Áë»Ø¹é¹À¼ÆÒªÓÅÓÚ×îÐ¡¶þ³Ë¹À¼Æ¡£

## ±¾ÕÂÐ¡½á

ÕâÒ»ÕÂÔÚ½éÉÜÁË±¾ÎÄÑÐ¾¿µÄ¿ÎÌâ±³¾°ÒÔ¼°ÑÐ¾¿¿ÎÌâµÄÀíÂÛÒâÒåºÍÊµ¼ÊÒâÒåÖ®Íâ£¬»¹½éÉÜÁË¹úÄÚÍâ¶Ô´ËµÄÑÐ¾¿ÏÖ×´¡£½éÉÜÍêÏàÓ¦µÄÄÚÈÝÖ®ºó¶Ô±¾ÕÂµÄÕûÌå½á¹¹×öÁËÒ»¸ö¼òµ¥µÄ½éÉÜ¡£×îºó£¬¶ÔÓÚ±¾ÎÄÐèÒªµÄÏà¹Ø±³¾°ÖªÊ¶Ò²½øÐÐÁË¼òÒªËµÃ÷¡£Í¨¹ý½éÉÜ×îÐ¡¶þ³Ë¹À¼ÆºÍÁë»Ø¹é¹À¼ÆÕâÁ½ÖÖ²ÎÊý¹À¼ÆµÄ·½·¨£¬Òý³öÎÒÃÇºóÐøÐèÒªÑÐ¾¿µÄÄÚÈÝ¡£

# Lasso»Ø¹éÄ£ÐÍ

## Lasso»Ø¹éµÄ±³¾°

Í¨¹ýÐ÷ÂÛµÄÏà¹ØÖªÊ¶½éÉÜ£¬ÎÒÃÇÁË½âµ½¶ÔÓÚÒ»°ãÏßÐÔ»Ø¹é·½³Ì£¬£¬×îÐ¡¶þ³Ë¹À¼ÆµÃ³öµÄ¹À¼ÆÁ¿¾ßÓÐÒÔÏÂÓÅµã:(1) ·½·¨¼òµ¥Ò×ÐÐ£¬¼ÆËã²¢²»¸´ÔÓ.(2)ÔÚ²ÎÊý¹À¼ÆÁ¿ÎÞÆ«µÄ»ù´¡ÉÏ¾ù·½Îó²î×îÐ¡¡£ÓÚ´ËÍ¬Ê±£¬È±µãºÜÃ÷ÏÔ£º(1) Èç¹û¾ØÕó ²»ÊÇÁÐÂúÖÈ£¬Ôò ½á¹û²»Î¨Ò»;(2) µ±×Ô±äÁ¿Ö®¼ä´æÔÚ¸ß¶ÈÏà¹ØÐÔÊ±£¬»áµ¼ÖÂÔö´ó£¬´Ó¶øµ¼ÖÂÔö´ó¡£

Ãæ¶ÔÕâÖÖÇé¿ö£¬Ñ§ÕßÃÇÌá³öÁË³Í·£º¯Êý£¬ÕûÌåË¼Â·ÊÇÍ¨¹ýÎþÉüÒ»¶¨µÄÎÞÆ«ÐÔ£¬½ø¶ø½µµÍ¹À¼ÆÁ¿ÕûÌåµÄ¾ù·½Îó²î£¬½ø¶øÌá¸ßÄ£ÐÍÕûÌåµÄÔ¤²âÎó²î¡£ÆäÖÐµäÐÍµÄ´ú±í¾ÍÊÇÁë»Ø¹é¹À¼Æ£¬£¬Áë»Ø¹éµÄ½¨Á¢ÓÐÐ§µÄ½â¾öÁË×îÐ¡¶þ³Ë¹À¼ÆÖÐµ±×Ô±äÁ¿´æÔÚ¸ß¶ÈÏà¹ØÊ±µ¼ÖÂÔö´óµÄÇé¿ö¡£²¢ÇÒÔÚÊ±£¬Áë»Ø¹é¹À¼Æ½á¹ûÒ»°ã¶¼ÓÅÓÚ×îÐ¡¶þ³Ë¹À¼Æ¡£¾¡¹ÜËüµÄÔ¤²â×¼È·ÐÔ½ÏºÃ£¬µ«ÔÚ´ó¶àÊýÇé¿öÏÂ£¬Ëü²»ÄÜ½«»Ø¹éÏµÊýÊÕËõµ½0£¬µ¼ÖÂ»Ø¹é·½³ÌÖÐµÄ±äÁ¿ÊýÄ¿ÈÔÈ»½Ï¶à£¬Õâ²»ÀûÓÚÎÒÃÇÎÒÃÇÕÒµ½ºÍÔ¤²â½á¹ûÏà¹ØÐÔ×î¸ßµÄ±äÁ¿¡£

Ëæºó£¬ÎªÁË½â¾öÕâ¸öÎÊÌâ£¬Breiman(1995)ÄêÌá³öÁËnon-negative garrotte(¼ò³ÆÎªNNG)£¬Æä¶¨ÒåÈçÏÂ

ÆäÖÐ£¬Ëæ×ÅµÄ²»¶ÏÔö´ó£¬ÖÐ²¿·ÖÔªËØ±»Ñ¹ËõÖÁ0£¬´Ó¶øµÃ³ö¡£ÔÚÔ­ÂÛÎÄÖÐ£¬ÓÉÓÚ³õÊ¼ÊÇÍ¨¹ýÆÕÍ¨×îÐ¡¶þ³Ë¹À¼Æ·½·¨µÃµ½µÄ£¬Òò´Ë×îÐ¡¶þ³Ë¹À¼Æ´æÔÚµÄÈ±µãÔÚNNGÖÐÒÀÈ»´æÔÚ¡£µ±×Ô±äÁ¿¸öÊý´óÓÚÑù±¾ÊýÊ±£¬ÓÉÓÚ ²»ÎÈ¶¨£¬¹ÊÒ²²»ÎÈ¶¨.µ«ÊÇºóÀ´ÆäËûÑÐ¾¿ÈËÔ±Ö¤Ã÷£ºÓÃÆäËû³õÊ¼¹À¼Æ(ÈçLasso¡¢Áë»Ø¹é¡¢µ¯ÐÔÍø)Ê±£¬NNG¾ßÓÐ·Ç³£ºÃµÄÐÔÖÊ¡£

NNGµÄµ®ÉúÓë±¾ÎÄÑÐ¾¿µÄÖØµãLasso¹ØÏµ½ôÃÜ£¬ÕýÊÇBreimanµÄÂÛÎÄ¸ø1996 ÄêµÄTibshirani µÄÂÛÎÄÌá¹©ÁËLasso Áé¸Ð¡£

## Lasso»Ø¹éµÄ¶¨Òå

¼Ù¶¨ÎÒÃÇÓÐÊý¾Ý,ÆäÖÐ ×÷Îª½âÊÍ±äÁ¿£¬×÷ÎªÏìÓ¦±äÁ¿¡£¶ÔÓÚÒ»°ãµÄÏßÐÔ»Ø¹é¶øÑÔ£¬ÎÒÃÇ¼Ù¶¨¹Û²â¶ÔÏóÊÇ¶ÀÁ¢µÄ£¬²¢ÇÒ½âÊÍ±äÁ¿ÒÑ¾­½øÐÐ¹éÒ»»¯£¬¼´¡£ Èç¹û²»½øÐÐ¹éÒ»»¯£¬ÔòLassoµÃµ½µÄ½á¹û»áÊÜµ½¾ßÌå²âÁ¿µ¥Î»µÄÓ°Ïì¡£²¢ÇÒÎªÁË·½±ãÆð¼û£¬ÎÒÃÇ¼Ù¶¨ÏìÓ¦±äÁ¿ÒÑ¾­½øÐÐÖÐÐÄ»¯£¬¼´¡£

Áî,ÔòLasso µÄ¶¨ÒåÎª

ÆäÖÐ()³ÆÎªµ÷ºÍ²ÎÊý£¬µ÷ºÍ²ÎÊýµÄ´óÐ¡¿ØÖÆ×Å»Ø¹éÏµÊý×ÜÌåµÄ´óÐ¡¡£ÈôÁî, Ôò¡£ ¾Í»áÊ¹Ò»Ð©»Ø¹éÏµÊýËõÐ¡²¢×îÖÕÇ÷ÏòÓÚ0£¬Ò»Ð©½ÏÐ¡µÄÏµÊýÉõÖÁÖ±½ÓËõÐ¡ÖÁ0¡£ÖµÔ½´ó£¬¸÷¸öÏµÊýµÄÈ¡Öµ·¶Î§¾ÍÔ½´ó£¬Ä£ÐÍ´Ó¶ø¸üÓÐÄÜÁ¦ÄâºÏÑµÁ·Êý¾Ý¡£·´Ö®£¬ÖµÔ½Ð¡£¬ÏµÊýµÄÈ¡Öµ·¶Î§Ô½Ð¡£¬Ä£ÐÍ´Ó¶ø¸ü¼ÓÏ¡Êè£¬ÓÐ¸üºÃµÄ¿É½âÊÍÐÔ¡£Òò´Ë£¬ÐèÒªÕÒµ½ºÏÊÊµÄt Öµ£¬ÔÚÕâÁ½ÖÖÇé¿öÏÂÕÒµ½Ò»ÖÖÆ½ºâ¡£

ÓÉÓÚÏÈÇ°ÎÒÃÇ¼Ù¶¨ÒÑ¾­½øÐÐÖÐÐÄ»¯£¬Òò´Ë¿ÉÔÚLassoÓÅ»¯ÖÐÊ¡ÂÔ½Ø¾à¡£ Èç¹ûÎ´½øÐÐÖÐÐÄ»¯´¦Àí£¬Ôò·ÇÖÐÐÄ»¯Êý¾ÝµÃµ½µÄ½Ø¾à ¿ÉÍ¨¹ýÏÂÃæµÄ¹«Ê½À´¼ÆËã

ÆäÖÐºÍ±íÊ¾Ñù±¾¾ùÖµ¡£Òò´Ë£¬±¾ÎÄ»áºöÂÔLasso µÄ½Ø¾à¡£ Òò´Ë¹«Ê½[[2.2]](#X0d5f5f3d469570bf8d9559b0daabada2e4fadfc)×ª»¯Îª

Í¬Ê±£¬ÎªÁË·½±ãºóÐøÌÖÂÛ£¬¹«Ê½[[2.4]](#X8a16af997b52cb888232ab5d79a527b0716561c)Ò»°ã¸ÄÐ´³ÉÀ­¸ñÀÊÈÕÐÎÊ½£¬ÈçÏÂËùÊ¾

ÆäÖÐ¡£ÉÏÊ½Ö®ËùÒÔ±»³ÆÎªÀ­¸ñÀÊÈÕÐÎÊ½£¬ÊÇÒòÎª¸ù¾ÝÀ­¸ñÀÊÈÕ¶ÔÅ¼ÐÔÖÊ¿ÉÖª£¬Óë ÊÇÒ»Ò»¶ÔÓ¦µÄ¹ØÏµ¡£ÏÂÃæ¼òµ¥µÄÖ¤Ã÷µ± Ê±£¬ÇÒXÊÇÕý½»¾ØÕóÊ±£¬ ÓëÒ»Ò»¶ÔÓ¦¡£²»¹ýÔÚ´ËÖ®Ç°ÒýÈëÈíãÐÖµº¯ÊýµÄ¸ÅÄî£¬Æä¶¨ÒåÈçÏÂ

ÆäÖÐ,Îª·ûºÅº¯Êý£¬µ±Ê±£¬,µ± Ê±,,Òò´ËÈíãÐÖµº¯ÊýµÈ¼ÛÓÚ

ÔÚÂú×ãKKTµÄÌõ¼þÏÂ£¬ ÊÇ×îÓÅ½âµ±ÇÒ½öµ±

ÆäÖÐ±íÊ¾µÄ´ÎÌÝ¶È¡£¶ÔÓÚ¾ø¶ÔÖµº¯Êý£¬Æä´ÎÌÝ¶ÈµÄÐÎÊ½Îª,¼´µ± Ê±£¬;,¡£µ±ÊÇÕý½»¾ØÕóÊ±£¬, Ôò ÏÔÈ»£¬ºÍ¹¹³ÉÒÔÏÂ·½³Ì×é

²»Ê§Ò»°ãÐÔ£¬¼ÙÉè¾ùÎªÕýÊý£¬ÔòÈçÏÂ·½³Ì×é

½âÉÏÊ½¿ÉµÃ

Ö¤±Ï¡£

ÑÜÉúÖÁÒ»°ãÇé¿ö

Í¨¹ýÉÏÊö¿ÉÖªµ±Ô½´ó£¬ÔòÔ½Ð¡£¬¼´ÏµÊýÔ½Ç÷½üÓÚ0,Ä£ÐÍ´Ó¶ø¸ü¼ÓÏ¡Êè¡£·´Ö®Ô½Ð¡£¬ÔòÔ½´ó£¬¼´ÏµÊýÔ½²»Ç÷½üÓÚ0£¬Ä£ÐÍ´Ó¶ø²»³ÊÏÖÏ¡ÊèÐÔ¡£

## Lasso»Ø¹éµÄÇó½â

´ÓLassoµÄ¶¨Òå±ã¿ÉÒÔ¿´³ö£¬Çó½â¹À¼ÆÁ¿ÊÇÒ»¸öÍ¹ÓÅ»¯ÎÊÌâ£¬ÊµÖÊÉÏÊÇ½âÒ»¸ö´ø²»µÈÊ½Ô¼ÊøµÄ¶þ´Î¹æ»®ÎÊÌâ(Quadratic program,QP)¡£ Òò´Ë£¬ÓÐÐí¶àQP ·½·¨¿ÉÒÔÓÃÀ´Çó½âLasso¡£Tibshirani(1996) µÄÎÄÕÂÖÐ¸ø³öÁËÒ»¸öLasso Ëã·¨£¬µ«ÊÇÓÉÓÚLasso Ëã·¨ÖÐ´æÔÚ×ÅºÜ´óµÄÏÞÖÆ¡£ Efron(2004)Ìá³öÁËÒ»¸ö×îÐ¡½Ç»Ø¹éËã·¨£¬¸ÃËã·¨»á½«Õû¸ö½âµÄÂ·¾¢×÷Îª³Í·£²ÎÊýµÄÒ»¸öº¯Êý£¬µ«Ëü²¢²»ÊÊºÏ´ó¹æÄ£ÎÊÌâ£¬½ø¶øFriedman(2007)ºÍWu(2008) Ê×ÏÈ½«×ø±êÏÂ½µ·¨ÔËÓÃµ½Lasso Ä£ÐÍÖÐ£¬ÆäÔÚ½â¾ö¸ßÎ¬Êý¾ÝºÍ´ó¹æÄ£Ñù±¾ÖÐÆð×ÅÖÁ¹ØÖØÒªµÄ×÷ÓÃ¡£±¾½Ú½«¼òÒª½éÉÜÉÏËßËã·¨µÄË¼ÏëºÍ²½Öè¡£

### LassoËã·¨

Ê×ÏÈ¹Ì¶¨,¹«Ê½(2.4)¿ÉÒÔ±íÏÖÎª¾ßÓÐ¸ö²»µÈÊ½Ô¼ÊøµÄ×îÐ¡¶þ³Ë¹À¼Æ£¬¼òµ¥À´Ëµ¾ÍÊÇ¶ÔÓ¦ÓÚ¸öÏµÊýµÄÖÖ×éºÏ¡£Lawson¡¢Hansen(1974) Ìá³öÁË½âÂú×ãÏßÐÔ²»µÈÊ½ µÄ×îÐ¡¶þ³Ë¹À¼ÆµÄ·½·¨¡£µ«ÊÇ¸Ã·½·¨ÔÚLasso ÖÐÄÑÒÔÖ±½ÓÔËÓÃ£¬µ«ÊÇ¿ÉÒÔÍ¨¹ýÖð²½¼ÓÈëÔ¼ÊøÌõ¼þÀ´½â¾ö£¬×îÖÕÕÒµ½Ò»¸ö¿ÉÐÐµÄ£¬Âú×ãKKTÌõ¼þµÄ½â£¬¾ßÌåËã·¨ÈçÏÂ£º

1. Áî £¬ÕâÀïµÄ Í¨³£ÊÇ ¹À¼ÆÁ¿¡£
2. ¼ÆËã £¬ÕâÀï¡£
3. ÑéÖ¤ÊÇ·ñÂú×ã, ÈôÂú×ãÔòÍ£Ö¹£¬¼´Îª½â£¬·ñÔò¼ÌÐø¡£
4. Áî ¼ÓÈëµ½ÖÐ£¬¼´Áî£¬È»ºó·µ»Ø²½¡£

Õâ¸öËã·¨ÓÅµãÔÚÓÚÖð²½¼ÓÈëÔ¼ÊøÌõ¼þ£¬Ê¹¼ÆËã½ÏÎª¼òµ¥¡£µ«ÊÇÈ±µãÒ²ºÜÃ÷ÏÔ£º(1) µ±×Ô±äÁ¿½Ï¶àÊ±£¬¸ÃËã·¨µÄËÙ¶È½ÏÂý£¬¶ÔÓÚ´óÊý¾Ý²¢²»ÊÊÓÃ¡££¨2£©µ± ÁÐÏòÁ¿²»ÂúÖÈÊ±£¬¸ÃËã·¨²»ÊÊÓÃ¡£

### ×îÐ¡½Ç»Ø¹éËã·¨

¸ÅÀ¨À´Ëµ×îÐ¡½Ç»Ø¹éËã·¨(Least Angle Regression,¼ò³ÆLARS)ÓëÏòÇ°Ñ¡Ôñ·¨(Forward Selection) ÀàËÆ¡£ÏòÇ°Ñ¡Ôñ·¨ÊÇÍ¨¹ýÃ¿´ÎÔö¼ÓÒ»¸ö±äÁ¿À´½¨Á¢Ä£ÐÍ£¬¼´Ã¿²½¶¼»áÌô³ö×îºÃµÄ±äÁ¿·Åµ½Ä£ÐÍÖÐ£¬È»ºóÓÃÄ£ÐÍÖÐËùÓÐµÄ±äÁ¿À´½øÐÐ×îÐ¡¶þ³ËÄâºÏ¡£¶ø×îÐ¡½Ç»Ø¹é²ÉÓÃÁËÏàÍ¬µÄ²ßÂÔ£¬µ«Ö»»á¾¡¿ÉÄÜ¶àµØÊäÈëÊÊºÏÄ£ÐÍµÄ±äÁ¿¡£µÚÒ»²½µÃµ½ÓëÊä³ö×îÏà¹ØµÄ±äÁ¿¡£LARS»á²»¶Ï³¯×îÐ¡¶þ³ËÖµÒÆ¶¯¸Ã±äÁ¿µÄÏµÊý£¨Õâ»áÈÃËüÓë²Ð²îµÄÏà¹ØÐÔÔÚ¾ø¶ÔÖµÉÏ¼õÉÙ£©£¬¶ø²»ÊÇÄâºÏ¸Ã±äÁ¿¡£Ò»µ©±ðµÄ±äÁ¿Óë²Ð²îµÄÏà¹ØÐÔ´ïµ½¸ÃÖµ£¬Õâ¸ö¹ý³Ì¾Í»áÔÝÍ££¬²¢½«µÚ¶þ¸ö±äÁ¿¼Óµ½»î¶¯¼¯ÖÐ£¬°´ÏàÍ¬·½Ê½ÐÞ¸ÄËüÃÇµÄÏµÊý£¬²¢½«Ïà¹ØÏµÊý°ó¶¨£¬¹²Í¬¼õÉÙ¡£µ±Ä£ÐÍÖÐµÄËùÓÐ±äÁ¿¶¼½øÐÐÁËÒ»´Î×îÐ¡¶þ³ËÄâºÏºó£¬Õâ¸ö¹ý³Ì»áÍ£Ö¹¡£¾ßÌåËã·¨ÈçÏÂ£º

1. Ê×ÏÈ¶ÔÑµÁ·Ñù±¾½øÐÐ¹éÒ»»¯£¬Ê¹ÆäÑù±¾¾ùÖµÎª0¡£³õÊ¼²Ð²îÎª¡£
2. ÕÒµ½Óë×îÏà¹ØµÄ£¬¼´, ¶¨ÒåºÍÒ»¸öÓÉµ¥±äÁ¿¹¹³ÉµÄ¾ØÕó¡£
3. ¶ÔÓÚ , ½øÐÐÒÔÏÂ²Ù×÷£º
   1. ¶¨Òå×îÐ¡¶þ³Ë·½Ïò£¬¶ÔÓÚp Î¬ÏòÁ¿, Ê¹£¬ÆäËûÔªËØ¶¼Îª0¡£
   2. ÒÔ·½Ïò£¬´Ó¿ªÊ¼£¬³¯×ÅËüÃÇµÄ×îÐ¡¶þ³Ë½âÒÆ¶¯ÏµÊý,ÆäÖÐ, ¡£½øÒ»²½µÃµ½ÐÂµÄ²Ð²î¡£
   3. ¹Ø×¢,Éèµ±¸ö±äÁ¿ÈÃ ´ïµ½×î´ó£¬ÔòÓ¦¸Ã½« ¼ÓÈë£¬ÇÒÁî¡£
   4. ÈÃ,,¡£
4. ·µ»ØÐòÁÐ

½âÊÍÏÂ¸ÃËã·¨µÚÈý²½ÖÐµÄ£¬Èç¹û£¬ÔÚ¾­¹ý²½Ö®ºó£¬LAR»áµÃµ½²Ð²îÎª0 µÄ½â¡£ÊÂÊµÉÏ£¬´ÓËã·¨²½ÖèÀ´¿´LARÔÚÔËËãÉÏÊÇ½Ú¼óµÄ£¬Æä¼ÆËãµÄÏûºÄÓëÍ¨³£µÄ×îÐ¡¶þ³Ë¹À¼ÆÊÇÒ»ÑùµÄ¡£µ«ÊÇ¶ÔÓÚ½âÊÍ±äÁ¿ÊÇµÄÇéÐÎ£¬¸Ã·½·¨×î¶àÑ¡µÄ±äÁ¿¸öÊýÎª,ÓÃ¸ÃËã·¨ÍùÍù»áµÃµ½¹ýÓÚÏ¡ÊèµÄÄ£ÐÍ¡£

### ×ø±êÏÂ½µ·¨

Wu(2008)Ê×´ÎÌá³ö×ø±êÏÂ½µ·¨ÔÚLassoÖÐµÄÓ¦ÓÃ£¬Æä±¾ÖÊÊÇÒ»ÖÖµü´úËã·¨£¬ÔÚµ¥¸ö×ø±ê·½ÏòÖ´ÐÐµ½µÄµü´ú£¬È»ºóÔÚ¸Ã×ø±ê·½ÏòÉÏÇóµ¥±äÁ¿×îÐ¡Öµ¡£¸ü×¼È·µÄ½²£¬Èç¹ûµÚ ´Îµü´úÑ¡ÔñÁË×ø±ê¡£¾ßÌåµÄËã·¨¹ý³ÌÈçÏÂ£º

1. Ê×ÏÈ¶ÔÓÚËæ»úÈ¡Ò»¸ö³õÖµ£¬¼ÇÎª£¬±íÊ¾³õÊ¼ÂÖÊý¡£
2. ¶ÔÓÚµÚÂÖµÄµü´ú¡£ÎÒÃÇ´Ó¿ªÊ¼£¬µ½ ÎªÖ¹£¬ÒÀ´ÎÇó¡£¾ßÌå±í´ïÊ½Îª£º¡£ ÆäÖÐ,Òò´Ë×îÐ¡ÖµºÜÈÝÒ×Í¨¹ýÇóµ¼ÇóµÃ¡£
3. µ± ½µµÍµ½¹æ¶¨ãÐÖµ£¬µü´ú½áÊø£¬·ñÕßÖØ¸´2²½£¬¼ÌÐø´Îµü´ú¡£

ÔÚLasso»Ø¹éÇó½âÖÐ£¬µ±·Ç³£´óÊ±£¬ÇóÈ«ÌÝ¶È»òÕß´ÎÌÝ¶ÈÐèÒªÏûºÄ´óÁ¿µÄÊ±¼äºÍ´¢´æ¿Õ¼ä£¬¶ø×ø±êÏÂ½µ·¨¿ÉÒÔÍ¨¹ýÇó½âÄ¿±êº¯ÊýµÄ×Óº¯ÊýÀ´Çó½â×îÓÅ½â¡£ÕâÀïµÄ×Óº¯Êý¿ÉÒÔ¿´×öÊÇ×ø±êÏµÖÐµÄÄ³Ò»¸ö×ø±ê»òÄ³Ò»×é×ø±ê£¬±£³ÖÆäËûµÄ×ø±êÏµµÄ±äÁ¿²»±ä£¬Ã¿´Îµü´úÖ»ÓÅ»¯²½Öè2 ÖÐµÄÄ¿±êº¯Êý£¬Òò´ËºÍÇ°Á½ÖÖËã·¨±È½Ï£¬ËüµÄÔËÐÐËÙ¶È·Ç³£Ñ¸ËÙ£¬²¢ÇÒ×ø±êÏÂ½µ·¨ÒÑ¾­±»¹ã·ºÓ¦ÓÃÓÚLasso ´¦Àí´ó¹æÄ£Êý¾ÝÖÐ¡£

## Lasso»Ø¹éµÄÐÔÖÊ

ÔÚ¹ýÈ¥Ê®¼¸ÄêÖÐ£¬Ñ§ÕßÃÇÒÑ¾­µÃµ½ÁËLasso¹À¼ÆµÄÖî¶àÁ¼ºÃÐÔÖÊ£¬±¾½Ú¶ÔÕâÐ©ÐÔÖÊ½øÐÐÁË¹éÄÉ×Ü½á¡£

### ÄâºÏÖµµÄÎ¨Ò»ÐÔ

Tibshirani(2013)µÄÎÄÕÂÖÐÖ¸³ö£¬Èç¹ûÖÐµÄÊý¾ÝÈ¡×ÔÒ»¸öÁ¬Ðø¸ÅÂÊ·Ö²¼£¬Ôò¶ÔÓÚ£¬LassoÎÊÌâ£¨¹«Ê½2.5£©µÄÄâºÏÖµÊÇÎ¨Ò»µÄ¡£¼´Ê¹£¬ÕâÒ»µãÈÎÈ»³ÉÁ¢£¬¾¡¹ÜÈÎºÎLasso ½âµÄ·ÇÁãÏµÊý¸öÊý×î¶àÎª¡£

¶ÔÓÚ×îÐ¡¶þ³Ë¹À¼Æ¶øÑÔ£¬Èô¾ØÕó²»ÊÇÁÐÂúÖÈ£¬ÆäÄâºÏÖµÎ¨Ò»£¬µ«²ÎÊý¹À¼Æ±¾Éí²»Î¨Ò»¡£·ÇÂúÖÈÇéÐÎÔÚÊ±¿ÉÄÜ³öÏÖ¹²ÏßÐÔÎÊÌâ£¬ÔÚÊ±Ôò×Ü»á³öÏÖ¡£¶ÔÓÚºóÒ»ÖÖÇé¿ö,µÄ½âÓÐÎÞÏÞ¶à¸ö£¬ÕâÐ©½â¶¼»áÈÃÑµÁ·Îó²îÎªÁã¡£ÏÖÔÚ¿¼ÂÇÔÚµÄÇéÐÎÏÂ£¬¼ÙÉèÓÐÁ½¸öLasso½â ºÍ,¶ÔÓ¦µÄ×îÓÅ½âµÄÖµÎª,ÔòÓÐ¡£ ÏÂÃæ½øÐÐÖ¤Ã÷¡£

£¨1£©¼ÙÉèºÍÎªLassoÄ£ÐÍµÄ½â£¬ÇÒ¡£ ÒòÎª¶þÕß¶¼ÊÇ¸ÃÎÊÌâµÄ×îÓÅ½â£¬Òò´ËÎÒÃÇÖªµÀ,¶ÔÓÚÈÎÒâ ¶øÑÔ£¬¶¼Îª¸ÃÎÊÌâµÄ×îÓÅ½â¡£

£¨2£©ÒÑÖª£¬Ôò¡£¶ÔÓÚÈÎºÎ£¬ÓÐÒÔÏÂ²»µÈÊ½

ÕâÒâÎ¶×Å»ñµÃÁËÒ»¸ö±È ¸üÐ¡µÄÖµ£¬ÕâÃ÷ÏÔÊÇÃ¬¶ÜµÄ¡£

£¨3£©Í¨¹ý2²½£¬ÎÒÃÇÖªµÀ±ØÐë³ÉÁ¢£¬²ÅÄÜµÃµ½ÏàÍ¬µÄ¾ù·½Îó²î£¬Òò´Ë¸ù¾Ý³Í·£ÐÔµÄÍ¹ÐÔ£¬Ôò¶ÔÓÚÈÎºÎ,¡£ ÏêÏ¸µÄÖ¤Ã÷ÔÚTibshiraniµÄÂÛÎÄÖÐ¸ø³öÁËÕâ·½ÃæµÄ×Ü½á¡£²»¹ý¼ÆËãLassoµÄÊýÖµËã·¨Í¨³£ÓëËã·¨µÄ¾ßÌåÏ¸½ÚÓÐ¹Ø¡£²ÉÈ¡²»Í¬µÄËã·¨£¬³õÊ¼ÖµµÄÑ¡Ôñ»áÓ°Ïì×îÖÕµÄ½â¡£

### ¹À¼ÆÁ¿µÄÏàºÏÐÔ

ÔÚÐ÷ÂÛÖÐ±¾ÎÄ½éÉÜÁË×îÐ¡¶þ³Ë¹À¼ÆºÍÁë»Ø¹é¹À¼ÆµÄÒ»ÖÂÐÔ£¬ÄÇÃ´ÔÚLassoÖÐÐèÒªÊ²Ã´¸½¼ÓÌõ¼þ¿ÉÈ·±£²ÎÊýÏòÁ¿²îÖµ Ò²ÊÕÁ²ÓÚ0¡£Knight ºÍFU(2000) Í¨¹ýÑÐ¾¿Ö¤Ã÷ÁËÔÚ×Ô±äÁ¿¸öÊý ºÍÕæÊµ¹À¼ÆÁ¿ ²»±äµÄÇé¿öÏÂ£¬Lasso¹À¼ÆÁ¿¾ßÓÐÒ»ÖÂÐÔ¡£ÏÂÃæ¶ÔÆäÖ¤Ã÷£º

Ê×ÏÈÎÒÃÇ×ö³öÈçÏÂ¼ÙÉè£º(1)£¬ÇÒ·ÇÆæÒì»òÕßÁÐÂúÖÈ¡£(2)¡£(3)£¬¼´,ÇÒ±íÊ¾»áËæ×ÅÑù±¾Á¿ µÄ±ä»¯¶ø±ä»¯¡£

ÔÚÉÏÊöÌõ¼þÖÐ£¬µ±ÊÇi.i.dÇÒÊÇ¶þ½×¾ØÓÐÏÞ£¬Ôò, Òò´ËÇ°Á½¸öÌõ¼þºÜºÃÂú×ã¡£Ç°ÎÄÒÑ¾­Ëµ¹ýÎªÁËÈ·±£Ã»ÓÐÁ¿¸ÙµÄÓ°Ïì£¬ÒÑ¾­¶Ô ½øÐÐ¹éÒ»»¯£¬¶Ô ½øÐÐÖÐÐÄ»¯¡£Ôò

²¢ÇÒÒªÏëÉÏÊ½´æÔÚ×îÓÅ½â£¬ÔÚÂú×ãKKT µÄÌõ¼þÏÂ£¬ÊÇ×îÓÅ½âµ±ÇÒ½öµ±

½«Ìáµ½·½³Ì×ó±ß£¬Ôò

µ±Ê±,, ºÜÃ÷ÏÔ£¬¡£ Õâ¾ÍÊÇ×îÐ¡¶þ³Ë¹À¼ÆµÄÎÞÆ«ÐÔ¡£µ±Ê±

¹«Ê½Ç°°ë²¿·Ö¾ÍÊÇ×îÐ¡¶þ³Ë¹À¼ÆµÄÎÞÆ«ÐÔ£¬¶ø¹«Ê½µÄºó°ë²¿·Ö£¬ÓÉÓÚÏÈÇ°¼ÙÉè¹ý£¬Òò´Ë¡£Ö¤±Ï¡£Òò´ËLasso¹À¼ÆÁ¿ÔÚÉÏÊöÌõ¼þ³ÉÁ¢ÏÂÊÇÒ»ÖÂ¹À¼ÆÁ¿¡£

### ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ

¶ÔÓÚLasso¹À¼ÆÊÇ·ñÈçÕæÊµ»Ø¹éÏòÁ¿Ò»ÑùÔÚÏàÍ¬µÄÎ»ÖÃÓÐ·ÇÁãÏî¡£¾ßÌå¶øÑÔ£¬¼ÙÉèÕæÊµ»Ø¹éÏòÁ¿ÊÇÏ¡ÊèµÄ£¬Ä¿±êÊÇÕýÈ·µÄÕÒ³öÓëÕæÊµ»Ø¹éÏàÍ¬µÄ¾ßÓÐÏàÍ¬¼¯ºÏµÄ×îÓÅ½â¡£Õâ¸öÐÔÖÊ³ÆÎª±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ£¬Ò²¿ÉÒÔ½Ð×öÏ¡ÊèÐÔ¡£Æä¶¨ÒåÈçÏÂ

¼´µ±Ñù±¾Á¿ Ê±£¬Í¨¹ý¸Ã·½·¨Ñ¡Ôñ³öÕýÈ·×Ô±äÁ¿µÄ¸ÅÂÊÇ÷½üÓÚ1¡£ ÔÚ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔµÄ»ù´¡ÉÏ£¬ZhaoºÍYu£¨2006£©Ìá³öÁË·ûºÅÒ»ÖÂÐÔ£¬Æä¶¨ÒåÈçÏÂ

ÆäÖÐ£¬ÊÇ·ûºÅº¯Êý£¬ÏÔÈ»·ûºÅÒ»ÖÂÐÔÊÇÇ¿ÓÚ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ£¬Ç°Õß½öÒªÇó·ÇÁãÏµÊýµÄÎ»ÖÃÒ»ÖÂ£¬¶øºóÕßÒªÇó·ÇÁãÏµÊýµÄÎ»ÖÃºÍ·ûºÅ¾ùÒ»ÖÂ¡£ÓÚ´ËÍ¬Ê±£¬×÷ÕßÔÚ·ûºÅÒ»ÖÂÐÔµÄ»ù´¡ÉÏ£¬¸ù¾ÝÕýÔò»¯µÄÇ¿Èõ¶¨ÒåÁËÁ½ÖÖ·ûºÅÒ»ÖÂÐÔ¡£

£ºµ±´æÔÚ£¬ÇÒº¯Êý¶ÀÁ¢ ºÍ¡£

£º´æÔÚ¡£

Ç¿·ûºÅÒ»ÖÂÐÔÒâÎ¶×ÅÓÐ¿ÉÒÔÊ¹ÓÃÔ¤ÏÈÑ¡ºÃµÄÀ´ÊµÏÖLassoµÄ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ¡£¶øÈõ·ûºÅÒ»ÖÂÐÔÔò±íÊ¾¶ÔÓÚËæ»úµÄÕýÔò»¯²ÎÊýÐèÒª´æÔÚÒ»¸öÕýÈ·µÄ À´Ñ¡³öÕæÊµµÄÄ£ÐÍ¡£ºÜÃ÷ÏÔÇ¿·ûºÅÒ»ÖÂÐÔÈõÕß·ûºÅÒ»ÖÂÐÔµÄ¡£ÈçºÎ²ÅÄÜ´ïµ½ÉÏÊöµÄÒ»ÖÂÐÔ½á¹û£¬×÷ÕßÔÚÔ­ÎÄÖÐ¸ø³öÁËÏàÓ¦Ìõ¼þºÍÒªÇó¡£

¼Ù¶¨£¬;£¬ÇÒ ºÍµÄ¹æÄ£Îª,,¡£ Áî

ÔÚ¿ÉÄæµÄÇ°ÌáÏÂ£¬´æÔÚÒ»¸öÕýÏòÁ¿£¬×÷ÕßÌá³öÁË

ºÍ

Á½ÕßµÄ²»Í¬µãÔÚÓÚ×óÊ½½Ó½üÓë1Ê±£¬Èõ²»¿É´ú±íÌõ¼þºã³ÉÁ¢£¬¶øÇ¿²»¿É´ú±íÌõ¼þÔÚ¼«ÏÞ´¦²»´æÔÚ¡£½ÓÏÂÀ´±¾½Ú·Ö±ðÌÖÂÛÔÚ³£¹æÊý¾ÝºÍ¸ßÎ¬Êý¾ÝÇé¿öÏÂµÄ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ¡£

¶ÔÓÚ¹Ì¶¨ÇÒ,³£¹æÊý¾ÝÏÂ

1. ¼Ù¶¨£¬ÆäÖÐÊÇÕý¶¨¾ØÕó¡£
2. ¡£
3. Ç¿²»¿É±íÊ¾Ìõ¼þ¡£

µ±ÉÏÊöÌõ¼þÖ»ÓÐÇ°Á½Ìõ³ÉÁ¢Ê±£¬µ±ÇÒ½öµ±´æÔÚÊ¹µÃÈõ²»¿É´ú±íÌõ¼þ³ÉÁ¢ºó£¬Lasso Ä£ÐÍ·ûºÏÈõ·ûºÅÒ»ÖÂÐÔ£»µ±Ç¿²»¿É´ú±íÒ²³ÉÁ¢Ê±£¬LassoÄ£ÐÍ·ûºÏÇ¿·ûºÏÒ»ÖÂÐÔ£¬¶ÔÓÚ·ûºÏ,£¬ÇÒ, Ôò

Òò´ËÇ¿²»¿É±íÊ¾Ìõ¼þ³ÉÁ¢£¬ÄÇÃ´Lasso Ñ¡ÔñÕæÊµÄ£ÐÍµÄ¸ÅÂÊÒÔÖ¸Êý½Ó½üÓÚ1£¬ÇÒ´ÓÉÏÒ»Ð¡½Ú¿ÉÖª¶ÔÓÚÊ±£¬LassoÒ²¾ßÓÐÒ»ÖÂ¹À¼ÆÁ¿£¬Òò´ËÇ¿²»¿ÉÌõ¼þÔÊÐíÍ¬Ê±½øÐÐÒ»ÖÂµÄÄ£ÐÍÑ¡ÔñºÍ²ÎÊý¹À¼Æ¡£ÁíÒ»·½Ãæ£¬¼´Ê¹¶ÔÓÚÈõ·ûºÅÒ»ÖÂÐÔ£¬Èõ²»¿É´ú±íÌõ¼þÒ²ÊÇ±ØÒªµÄ¡£Òò´Ë£¬ÔÚ³£¹æÊý¾ÝÏÂ£¬Ç¿²»¿É´ú±íÌõ¼þÇ¿·ûºÅÒ»ÖÂÐÔÈõ·ûºÅÒ»ÖÂÐÔÈõ²»¿É´ú±íÐÔ¡£

¸ßÎ¬Êý¾ÝÇé¿öÏÂ£¬ÓÉÓÚ »áËæ×ÅµÄÔö³¤¶øÔö³¤£¬Òò´ËÁî, ²¢ÇÒ ºÍÒ²»áËæ×Å µÄÔö³¤¶ø±ä»¯¡£¼Ù¶¨´æÔÚ Ê¹µÃ

1. ¡£
2. ¡£
3. ¡£
4. ¡£
5. Ç¿²»¿É±íÊ¾Ìõ¼þ¡£

µ±ÉÏÊöÌõ¼þ¾ù³ÉÁ¢£¬ÇÒÂú×ã;Ê±¡£ÓÐ

Òò´Ë¶ÔÓÚ¸ßÎ¬Êý¾Ý¶øÑÔ£¬ÏëÒª´ïµ½·ûºÅÒ»ÖÂÐÔ²»½ö½öÐèÒªÇ¿²»¿É±íÊ¾Ìõ¼þ£¬»¹ÐèÒª¶ÔÔëÉù½øÐÐÏàÓ¦µÄÏÞÖÆ¡£

## ±¾ÕÂÐ¡½á

±¾ÕÂÖ÷Òª¶ÔLasso»Ø¹éÄ£ÐÍ½øÐÐÁËÒ»¸öÏêÏ¸µÄËµÃ÷Óë¸ÅÀ¨¡£Ê×ÏÈ¶ÔÓÚLasso»Ø¹éÄ£ÐÍµÄ±³¾°½øÐÐÁË¼òÒªµÄ½éÉÜ£¬ËµÃ÷Æä½¨Á¢µÄÔ­ÒòºÍ¹ý³Ì¡£Ö®ºó¸ø³öÁËLasso»Ø¹éµÄ¶¨Òå£¬²¢ÇÒ¾ÍLassoµÄÆÕÍ¨ÐÎÊ½ºÍÀ­¸ñÀÊÈÕÐÎÊ½Ò»Ò»¶ÔÓ¦µÄ¹ØÏµ½øÐÐÁË¼òµ¥µÄÖ¤Ã÷£¬ÓÚ´ËÍ¬Ê±ÒýÈëÁËÈíãÐÖµº¯ÊýµÄ¸ÅÄî¡£½ô½Ó×ÅÂÞÁÐÁËLassoÇó½âËã·¨µÄ¾ßÌåË¼ÏëºÍ²½Öè£¬ÆäÖÐ°üÀ¨Lasso Ëã·¨¡¢×îÐ¡½Ç»Ø¹éËã·¨ºÍ×ø±êÏÂ½µ·¨¡£×îºó£¬¶ÔÓÚLasso»Ø¹éÄ£ÐÍÖÐ¾ßÓÐµÄÁ¼ºÃÐÔÖÊ½øÐÐÁËÏàÓ¦µÄÖ¤Ã÷ºÍ¸ÅÊö£¬ÆäÖÐ°üÀ¨ÄâºÏÖµµÄÎ¨Ò»ÐÔ£¬¹À¼ÆÁ¿µÄÏàºÏÐÔºÍ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ£¨Ï¡ÊèÐÔ£©¡£

# »ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éµÄÀíÂÛÑÐ¾¿

±¾ÕÂ½«ÔÚLasso»Ø¹éÄ£ÐÍµÄ»ù´¡ÉÏÒýÈëÒ»¸ö¸üÒ»°ãµÄ»Ø¹éÄ£ÐÍ£¬¼´»ùÓÚÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éÄ£ÐÍ£¬²¢ÔÚ´ËÖ®ºóÌá³ö±¾ÎÄµÄºËÐÄÄÚÈÝ£¬¼´¼ÓÈëÏÈÑéÐÅÏ¢ºóµÄLasso»Ø¹éÄ£ÐÍ¡£Òò´Ë±¾ÎÄ½«Ê×ÏÈÔÚ±¾ÕÂµÚÒ»½Ú¸ø³öÏà¹ØµÄÖªÊ¶¸ÅÄî£¬Æä´ÎÔÚµÚ¶þ½Ú½éÉÜÁËÐÂÄ£ÐÍµÄ¶¨Òå£¬¼°ÆäÊ¹ÓÃ³¡¾°¡£×îºóÔÚµÚÈý½Ú¸ø³öÁËÐÂÄ£ÐÍµÄÇó½â·½·¨¡£

## Ïà¹ØÖªÊ¶¹¤×÷

### Ï¡Êè¿ò¼Ü

ÔÚ¹ýÈ¥¼¸ÄêÖÐ£¬Ï¡ÊèÐÔÒÑ¾­³ÉÎªÓ¦ÓÃÊýÑ§¡¢¼ÆËã»ú¿ÆÑ§ºÍµçÆø¹¤³Ì¸÷¸öÁìÓòµÄÒ»¸öÖØÒª¸ÅÄî¡£ÔÚÏ¡ÊèÐÅºÅ´¦ÀíÖÐ£¬ÎªÁËÊµÏÖÐí¶àÀàÐÍµÄÐÅºÅÖ»ÓÃ¼¸¸ö·ÇÁãÏµÊý±íÊ¾£¬RobertµÈ(2011) Ìá³öÁËÏ¡Êè¿ò¼ÜµÄ¸ÅÄî£¬²¢½øÐÐÁËÉîÈëÑÐ¾¿¡£Ï¡Êè¿ò¼ÜµÄ¸ÅÄîÈçÏÂ¡£

ÉèÊÇ¿Õ¼äÉÏµÄÒ»×é±ê×¼Õý½»»ù¡£Èç¹û¶ÔÃ¿¸ö£¬¶¼´æÔÚ£¬Ê¹µÃ ÖÐµÄ¿ò¼ÜÖÐµÄÃ¿¸öÔªËØ¶¼Âú×ã

ºÍ

ÄÇÃ´³Æ¿ò¼Ü ÊÇ½×Ï¡Êè¿ò¼Ü£¬ÆäÖÐ ±íÊ¾ ÖÐÔªËØµÄ¸öÊý£¬²¢¼Ç

ÏÔ¶øÒ×¼û£¬¡£Áî£¬ Óë£¬ÔòÓÐ

¼´Ï¡Êè¿ò¼Ü¿ÉÒÔ±íÊ¾Îª

ÆäÖÐ,Ò²±»³ÆÎª»ùÏµÊý£¬Îª±ê×¼Õý½»»ù¹¹³ÉµÄ¾ØÕó

### »ùÓÚÏ¡Êè¿ò¼ÜµÄLasso»Ø¹é

ÔÚ±¾½ÚÖÐ£¬Ê×ÏÈÎÒÃÇ»Ø¹ËÒ»°ãµÄ»Ø¹éÎÊÌâ

ÆäÖÐÊÇÒò±äÁ¿£¬ÊÇµÄ×Ô±äÁ¿¾ØÕó¡£ÎªÁËµÃµ½Ï¡ÊèµÄ, ºÍ¸ü¼Ó¾«×¼µÄÔ¤²âÎó²î¡£LassoÊÇµ±Ç°×îÁ÷ÐÐ¡¢×î»ù±¾µÄ·½·¨Ö®Ò»¡£ËüµÄÀ­¸ñÀÊÈÕÐÎÊ½Îª

»òÒÔ¶àÄ¿±êµÄ·½Ê½±íÊ¾

½ÓÏÂÀ´£¬ÎÒÃÇÒýÈëÉÏÒ»½Ú½éÉÜµÄÏ¡Êè¿ò¼Ü£¬½«ÕýÔò»¯ÖÐµÄ¿´³ÉÏ¡Êè¿ò¼ÜÖÐµÄ£¬¼ÓÈëÒ»¸ö ¾ØÕóÀ´´úÌæÔ­ÏÈµÄ£¬µ«Çø±ðÔÚÓÚ£¬ÕâÀïµÄ²»ÔÚÊÇÓÉ±ê×¼Õý½»»ù¹¹³ÉµÄ£¬¶øÊÇÓÉÒ»¸öÓÃ»§Ö¸¶¨µÄÏ¡Êè¾ØÕóÀ´Ìæ´ú¡£¿ÉÒÔ¿´³ÉÊÇÔÚLassoÄ£ÐÍ»ù´¡ÉÏ¸üÎªÒ»°ãµÄÐÎÊ½£¬Æä¶¨ÒåÎª

ÆäÖÐ£¬ÊÇÈÎÒâ¹¹ÔìµÄÏ¡Êè¾ØÕó£¬Æä×÷ÓÃÊÇÍ¨¹ýÏ¡Êè±àÂëÀ´ÏÞ¶¨ÕæÊµ Ö®¼äµÄ¹ØÏµ¡£ÏÂÃæ¾Ù³ö¼¸¸öÀý×ÓËµÃ÷¡£

**Example 3.1** (Lasso). *ºÜÃ÷ÏÔ£¬µ±Îªµ¥Î»¾ØÕóÊ±£¬Ô­Ê½¾Í±äÎª¾­µäµÄLasso Ä£ÐÍ¡£*

**Example 3.2** (Fused Lasso). *Tibshirani(2005)Ìá³öÁËFused Lasso£¬ÓÃÓÚ´¦ÀíÓÐÐò±äÁ¿Êý¾Ý½á¹¹£¬ÓÈÆäÉÃ³¤½â¾ö»ùÒò×éÊý¾ÝºÍÐÅºÅÊý¾Ý£¬Æä¾ßÌåÐÎÊ½ÈçÏÂ µÚÒ»¸ö³Í·£ÏîÊÇ³£¼ûµÄ·¶Êý£¬ÆäÄ¿µÄÓëLassoÄ£ÐÍÒ»ÖÂ£¬Ñ¹ËõÏµÊý Ç÷ÏòÓÚ0¡£ µÚ¶þ¸ö³Í·£ÏîÊÇÕë¶ÔÓÐÐòµÄÊý¾ÝÇé¿öÏÂ£¬´ÙÊ¹ÏàÁÚÏµÊýÇ÷ÓÚÒ»ÖÂ£¬¶øÇÒµ¼ÖÂÒ»Ð©ÏµÊýÒ»ÖÂ¡£½«ÉÏÊö¹«Ê½×ª»¯ÎªÏ¡Êè¿ò¼ÜÏÂÐÎÊ½£¬µ± ÔòFused Lasso¼´×ª»¯ÎªÏ¡Êè¿ò¼ÜÏÂµÄÐÎÊ½¡£ÆäÄÚÈÝÓëFused LassoÍêÈ«Ò»ÖÂ¡£*

**Example 3.3** (Clustered Lasso). *She(2010)Ìá³öÁËClustered Lasso,ÆäÖ÷ÒªÄ¿µÄÊÇ¼´±£³ÖÓÐLassoÄ£ÐÍµÄ±äÁ¿Ñ¡Ôñ£¬ÓÖÄÜ¹»ÀûÓÃ¾ÛÀàµÄË¼Ïë½«±äÁ¿½øÐÐ·Ö×é£¬Æä¾ßÌåÐÎÊ½ÈçÏÂ ½«ÉÏÊö¹«Ê½×ª»¯ÎªÏ¡Êè¿ò¼ÜÏÂÐÐÊ»£¬µ± ÆäÖÐÎª³É¶Ô²î·Ö¾ØÕó¡£ÐÎÊ½ÈçÏÂ ÆäÖÐ,ÎªÕæÊµµÄ¸öÊý¡£ ÁÐ¾ÙÁËËùÓÐ¿ÉÄÜµÄ³É¶Ô×éºÏ¡£µ±Ê±£¬¡£ Clustered LassoËäÈ»ÓëFused Lasso ÐÎÊ½ÉÏÏàËÆ£¬µ«Ëû²»ÐèÒª¶Ô»Ø¹éÌØÕ÷½øÐÐÅÅÐò£¬ÆäÊÔÍ¼×éÖ¯ÎÞÐòµÄÌØÕ÷²¢²úÉú½á¹ûÅÅÐò£¬Êµ¼ÊÉÏClustered LassoÊÇÒ»ÖÖ¼à¶½¾ÛÀà·½·¨£¬Í¬Ê±¿¼ÂÇx ºÍy À´ÎªÌØÕ÷Ñ°ÕÒºÏÊÊµÄ·Ö×é¡£*

×Ü¶øÑÔÖ®£¬¿É¶¨ÖÆµÄ±íÊ¾ÔÚ»Ø¹é·ÖÎöÖÐÌá³öµÄÏ¡ÊèÐÔÒªÇó£¬ÓÃÓÚ±íÊ¾ÕæÊµ Ö®¼äµÄÌØ¶¨¹ØÏµ¡£Æä±íÏÖÐÎÊ½¿ÉÒÔÊÇ¶àÖÖ¶àÑù£¬¶ø±¾ÎÄµÄÖ÷ÒªÑÐ¾¿·½Ïò¾ÍÊÇ»ùÓÚÕâÖÖÏ¡Êè¿ò¼ÜµÄÐÎÊ½£¬ÈÚºÏÏÈÑéÐÅÏ¢ÐÎ³ÉÒ»ÖÖÐÂµÄÌØ¶¨Ä£ÐÍ¡£

## »ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éµÄ¶¨Òå

ÉÏÒ»½ÚÖÐ±¾ÎÄÏêÏ¸½éÉÜÁË»ùÓÚÏ¡Êè¿ò¼ÜµÄLasso»Ø¹éÄ£ÐÍ£¬ÁË½âÁËÆä±È½Ï³£¼ûµÄ¼¸ÖÖÌØ¶¨ÐÎÊ½£¬µ«ÊÇÕâ¼¸¸öÄ£ÐÍ¾ùÃ»ÓÐÀûÓÃµ½ÏÈÑéÐÅÏ¢¡£Òò´Ë£¬±¾ÎÄÌá³öÁËÒ»ÖÖÐÂµÄLasso¸Ä½ø·½·¨£¬¼´½«ÏÈÑéÐÅÏ¢ÈÚÈëLassoÄ£ÐÍ£¬±¾ÎÄ½«Æä³ÆÎª»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso»Ø¹éÄ£ÐÍ¡£

Èç¹û°ÑÔÚ»Ø¹é·ÖÎöÖÐÌØÕ÷µÄ×ÔÓÐÊôÐÔ¿´×öÒ»¸öÕûÌå£¬ÄÇÃ´ÏÈÑéÐÅÏ¢¾ÍÊÇ¸ÃÕûÌåÖÐµÄÒ»²¿·Ö¡£ÎÒÃÇÔÚ½¨Á¢Lasso»Ø¹éÄ£ÐÍÊ±£¬ÓÐÊ±»áÌáÇ°ÖªµÀËüÄÚÔÚµÄÒ»Ð©ÊôÐÔ¼°Æä×÷ÓÃ¡£²¢ÇÒÏÈÑéÐÅÏ¢±¾Éí¾ÍÊÇ´æÔÚµÄ£¬ËùÒÔÓëËÑ¼¯Êý¾ÝÎÞ¹Ø£¬·´Ö®ÕýÈ·ËÑ¼¯Êý¾ÝÓ¦¸ÃÊÜµ½ÏÈÑéÐÅÏ¢ÏÞÖÆ£¬²¢ÇÒÂú×ãÏÈÑéÐÅÏ¢¡£µ«ÊÇÏÈÑéÐÅÏ¢²»ÄÜÓÃÓÚÃèÊöÑÐ¾¿¶ÔÏó£¬ËùÒÔÐèÒª½«ÌØÕ÷±¾Éí¾ßÓÐµÄÏÈÑéÐÅÏ¢Ó¦ÓÃµ½Ä£ÐÍ½¨Á¢ÖÐ£¬²¢Í¨¹ýÄ£ÐÍµÄ½á¹ûÀ´±íÏÖ³öÏÈÑéÐÅÏ¢¡£

ÏÖÔÚ¿¼ÂÇÏ¡Êè¿ò¼ÜÏÂµÄLasso»Ø¹é£¬ÎªÁË¸üºÃµÄ±í´ï£¬½«¹«Ê½([[3.9]](#Xeeca99fc63e5f271c42e25accff2693ca5625d7))¸ÄÎªÀ­¸ñÀÊÈÕÐÎÊ½±íÊ¾

ÓÚ´ËÍ¬Ê±ÎªÁË±ãÓÚÃèÊöÈÚºÏÏÈÑéÐÅÏ¢ºóµÄÐÂÄ£ÐÍ£¬±¾ÎÄÔÚ´Ë¾Ù¸öÀý×ÓÊ¹¶ÁÕß¸üºÃµÄÀí½â¡£¼ÙÉèÎÒÃÇ»ñµÃ²ÎÊý¾ßÓÐÈçÏÂÏÈÑéÐÅÏ¢£º3¸ö²»Í¬µÄ ÊÇÒ»ÖÂµÄ£¬, ²¢ÇÒ¡£ÎªÁËÂú×ãÕâÖÖÏÈÑé£¬ÎÒÃÇ¿ÉÒÔ¹¹Ôì°üº¬

ÒÔ±ãÔÚ»Ø¹éÄâºÏÖÐ¸üºÃµÄ»ñµÃÕæÊµµÄ¡£µ«ÊÇ¾ÍËã°üº¬ÉÏËßÐÐÏòÁ¿£¬ÈÔÈ»¿ÉÒÔ¹¹Ôì²»Í¬µÄ ¾ØÕó¡£µ«Ö÷ÒªÓÐÒÔÏÂÁ½ÖÖ½á¹¹¡£

¼ÙÉèÎÒÃÇµÄÏ¡Êè¿ò¼ÜÎª·½ÕóÊ±£¬Æä¾ßÌåÐÎÊ½ÈçÏÂ

¼ÙÉèÎÒÃÇµÄÏ¡Êè¿ò¼Ü²»Îª·½ÕóÊ±£¬¼´¹¹ÔìµÄÐÎÊ½ÀàËÆFused Lasso¡£

Òò´Ë£¬ÔÚºóÎÄµÄÇó½âºÍÊµÑéÖÐ£¬±¾ÎÄ»á¾ÍÁ½ÖÖ²»Í¬ÐÎÊ½µÄ¾ØÕó½øÐÐ·ÖÎö±È½Ï¡£²¢ÇÒ¾ÍÏÈÑéÐÅÏ¢¶øÑÔ£¬ÐÅÏ¢¸ßµÍÒ²»áµ¼ÖÂ½á¹ûµÄÓÅÁÓ¡£

## »ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éµÄÇó½â

Ê×ÏÈ£¬ÎÒÃÇÒý³öÉÏÒ»ÕÂµÄLasso»Ø¹éÄ£ÐÍ£¬¸ø¶¨,Ôò

ÔÚÂú×ãKKTµÄÌõ¼þ£¬ÊÇ×îÓÅ½âµ±ÇÒ½öµ±Âú×ã

Áî¡£Ôòµ± Ê±£¬; £¬£»,¡£ÒýÓÃÇ°ÎÄµÄÈíãÐÖµ·ûºÅ£¬Ôò

½«¹«Ê½([[3.10]](#X73fd4c20d1e3f11519142bec6a1ac2b0b3338e5))ÖØÐ´Îª

Õâ¾Í¼¤·¢ÁËµü´úµÄÉè¼ÆÀ´½â¾ö¹«Ê½([[3.9]](#Xeeca99fc63e5f271c42e25accff2693ca5625d7))

ÆäÖÐ±íÊ¾µü´ú´ÎÊý£¬³õÊ¼ÖµµÄÑ¡¶¨¿ÉÒÔÓÃËæ»ú±äÁ¿£¬Ò²¿ÉÒÔÑ¡ÓÃ×îÐ¡¶þ³Ë¹À¼Æ×÷Îª³õÊ¼Öµ½øÐÐµü´ú¡£µ± Ê±£¬ÎÒÃÇÄ¬ÈÏµü´ú½áÊø¡£¸Ãµü´ú¹ý³ÌÊÕÁ²µ½Ò»¸ö×îÓÅµã¡£´Ëµü´ú·½Ê½ÒÑ¾­ÒÔ²»Í¬µÄÐÎÊ½Ìá³ö£¬²¢±»ÔËÓÃÓÚ½â¾ö´óÊý¾ÝÎÊÌâ¡£

¼Ù¶¨Ï¡Êè¾ØÕóÁÐÂúÖÈ£¬Òò´Ë´æÔÚÁ½ÖÖÐÎÊ½£¬Ò»ÖÖÎª³£¼ûµÄ·½Õó£¬ÁíÒ»ÖÖÐÎÈç µÄ"ÊÝ"¾ØÕó¡£½è¼øLassoµÄÇó½â·½·¨£¬ÔÚÂú×ãKKT µÄÌõ¼þ£¬ÊÇ×îÓÅ½âµ±ÇÒ½öµ±Âú×ã

»òÕßÊÇ

ÆäÖÐ,Ö®ºó½«¹«Ê½Á½±ßÍ¬Ê±¼ÓÉÏ, Ôò¹«Ê½([[3.25]](#Xca8ee65a3fa3737f6b3f5b0c6055c0e09305294))¸ÄÐ´Îª

Áî, ½«¹«Ê½ ([[3.26]](#Xc43da0e7b24f84e7ce2298f6cf8c4c41d86a810)) ÖØÐ´Îª

ÈôÏëÑØÓÃµü´úµÄË¼ÏëÐèÒª½«ÒÆÖÁ¹«Ê½ÓÒ±ß£¬ÄÇ¾ÍÒªÇó´æÔÚÄæ¾ØÕó¡£ ºÜÃ÷ÏÔ,µ±Ï¡Êè¾ØÕóÎª"ÊÝ"¾ØÕóÊ±£¬¾ØÕóÃ»ÓÐ×óÄæ¡£¾ÍÏñfused Lasso Ò»Ñù£¬Òò´Ë±¾ÎÄÐèÒª¶ÔÏ¡Êè¾ØÕóµÄÁ½ÖÖ²»Í¬Çé¿ö½øÐÐ·ÖÀàÌÖÂÛ¡£

### Ï¡Êè¿ò¼ÜTÎª·½Õó

Ê×ÏÈ£¬ÎÒÃÇÖØÖÃ²ÎÊý£¬Áî,µ±Ï¡Êè¿ò¼ÜÎª·½ÕóÊ±£¬ ´æÔÚ¡£½ÓÏÂÀ´½«À­¸ñÀÊÈÕµÄÒ»°ãÐÎÊ½Lasso([[3.9]](#Xeeca99fc63e5f271c42e25accff2693ca5625d7)) ÖØÐ´Îª

Âú×ãKKTÌõ¼þ£¬ÊÇ×îÓÅ½âµ±ÇÒ½öµ±Âú×ã

ÎªÁË±ãÓÚÔÄ¶Á£¬Áî, ´øÈë¹«Ê½([[3.29]](#Xf42d9eff165ff903900a0f16da2e2d14bf3c5d9))µÃµ½ÓëLasso½üËÆµÄÇó½â·½Ê½

»òÕßÊÇ

Áî,Ôò

ÖØÐ´¹«Ê½([[3.31]](#X9dcc1ab6de1c8860ed2dc6936960960658ac4ee))

Í¬ÀíÑØ×Åµü´úËã·¨µÃ³ö

²¢ÇÒ×îÖÕµÃ³ö

ÔÚÏ¡Êè¿ò¼ÜÎª·½ÕóÊ±£¬Çó½âËã·¨Óë±ê×¼LassoµÄÇó½âËã·¨Ã»ÓÐÌ«´óÇø±ð¡£ÆäËã·¨²½ÖèÈçÏÂ£º

1. ³õÊ¼»¯²ÎÊý£¬ÉèÖÃ³õÊ¼¡£
2. ¿ªÊ¼µü´ú:
   * Í¨¹ý¹«Ê½([[3.35]](#X004afeeb4513c0ad40862169cb66b8dd0f2b254))¸üÐÂ ºÍ¡£
   * Èç¹û ×ã¹»Ð¡£¬Í£Ö¹µü´ú¡£
   * ·ñÔò£¬Áî:¼ÌÐøÏÂÒ»´Îµü´ú¡£

### Ï¡Êè¿ò¼ÜTÎª"ÊÝ"¾ØÕó

¸Õ²Å½éÉÜÍêÏ¡Êè¿ò¼ÜÎª·½ÕóµÄÇó½â·½·¨£¬½ÓÏÂÀ´¿´¿´µ±Ï¡Êè¿ò¼ÜÎª"ÊÝ"¾ØÕóµÄ½â·¨¡£Í¬Ñù£¬ÎÒÃÇÖØÖÃ²ÎÊý£¬ÒýÈëÂú×ã£¬¼Ù¶¨µÄ·Ö½âÎª£¬ÎÒÃÇ¿ÉÒÔµÃ³ö¡£Òò´Ë¹«Ê½([[3.28]](#X059a3a08cb083fb14dffcd2d524cbd08ab9030c))µÈ¼ÛÒÔÏÂµÄLassoÄ£ÐÍ

Âú×ãKKTÌõ¼þ£¬ÊÇ×îÓÅ½âµ±ÇÒ½öµ±Âú×ã

Õ¹¿ª¹«Ê½([[3.37]](#X2cc8874a54d8d40b341d84839c1b2984efc0aba))

»òÕßÊÇ

Áî,Ôò

ÑØ×Åµü´úÕ¹¿ª

ÇÒ

ÔÚÄ³Ð©ÎÂºÍÌõ¼þÏÂ£¬¸ÃËã·¨ÊÕÁ²¡£ÆäËã·¨²½ÖèÈçÏÂ£º

1. ³õÊ¼»¯²ÎÊý£¬ÉèÖÃ³õÊ¼ µÈ¡£
2. ¿ªÊ¼µü´ú:
   * Í¨¹ý¹«Ê½([[3.42]](#X1f26c3dcb33cbdff7d1322ccba582f464286bc5))¸üÐÂ ºÍ¡£
   * Èç¹û ×ã¹»Ð¡£¬Í£Ö¹µü´ú¡£
   * ·ñÔò£¬Áî:¼ÌÐøÏÂÒ»´Îµü´ú¡£

ÔÚµÚËÄ½ÚÖÐ£¬±¾ÎÄ»á½«ÏÈÑéÐÅÏ¢·ÅÈëÏ¡Êè¿ò¼ÜÖÐ£¬±È½ÏÁ½ÖÖËã·¨ºÍÔ­Ê¼LassoÄ£ÐÍÖ®¼äµÄÓÅÁÓ¡£

## »ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso »Ø¹éµÄÐÔÖÊ

### ¹À¼ÆÁ¿µÄÏàºÏÐÔ

ÔÚµÚ¶þ½ÚÖÐ±¾ÎÄ½éÉÜÁËLasso»Ø¹é¹À¼ÆÁ¿µÄÒ»ÖÂÐÔ£¬ÄÇÃ´¶ÔÓÚÏ¡Êè¿ò¼ÜÏÂµÄLasso »Ø¹é£¬ÊÇ·ñ¾ßÓÐÏàÍ¬µÄÐÔÖÊ£¬ÏÂÃæ¶ÔÆä½øÐÐÖ¤Ã÷¡£

Ê×ÏÈÎÒÃÇ×ö³öÈçÏÂ¼ÙÉè£º(1)£¬ÇÒ·ÇÆæÒì»òÕßÁÐÂúÖÈ¡£(2)¡£(3)¡£

ÊÇÏ¡Êè¿ò¼ÜÏÂµÄLasso»Ø¹éµÄ×îÓÅ½âµ±ÇÒ½öµ± Âú×ã

µÈ¼ÛÓÚ£¬

»òÕßÊÇ£¬

ÓÉLasso¹À¼ÆÁ¿µÄÏàºÏÐÔºÍ¼ÙÉèÌõ¼þ¿ÉÖª£º£¬¡£ ºÜÃ÷ÏÔÒ»ÖÂÐÔÊÇÒ»¸öºÜÈõµÄÒªÇó£¬¶ÔÓÚºÍÃ»ÓÐÈÎºÎÏÞÖÆ£¬¿ÉÒÔºÜÇáÒ×µÄÍ¨¹ýÑ¡ÔñºÏÊÊµÄ À´ÊµÏÖ¡£Òò´ËµÃ³ö½áÂÛ£¬Ö»Òª£¬Ôò, ²¢ÇÒ¡£

### ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ

ÔÚÇ°Ò»½ÚÖÐ±¾ÎÄÏêÏ¸µÄ½éÉÜÁË±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔºÍ·ûºÅÒ»ÖÂÐÔ£¬²¢ÇÒºóÕßÊÇÃ÷ÏÔ¸ßÓÚÇ°ÕßµÄ£¬ÎªÁËÄÜ¹»ÓÐÐ§µÄÑ¡Ôñ³öÔÚÕæÊµ¾ßÓÐÏàÍ¬Î»ÖÃµÄ·ÇÁãÏî£¬Òò´ËÖ»ÒªËµÃ÷Ï¡Êè¿ò¼ÜÏÂµÄLasso »Ø¹éÄ£ÐÍ¾ßÓÐ·ûºÅÒ»ÖÂÐÔ¾ÍÄÜ±íÊ¾Æä¾ßÓÐÏ¡ÊèÐÔ£¬¼´ÐèÒªÖ¤Ã÷¡£

ÆäÊµºÜÃ÷ÏÔ£¬¹À¼ÆÁ¿µÄÏàºÏÐÔ¾ÍËµÃ÷ÁË±äÁ¿Ñ¡ÔñµÄÒ»ÖÂÐÔ¡£¼òµ¥À´Ëµ£¬¶ÔÓÚ ¶øÑÔ£¬ÎÒÃÇ´ÓÉÏÒ»½ÚÄÜµÃ³ö, ²¢ÇÒ¡£ÄÇÃ´¶ÔÓÚÇ°ÎÄÒýÈëµÄ·ûºÅÑ¡ÔñÐÔ£¬Ôò¿ÉÒÔºÜÇáÒ×µÄËµÃ÷¡£

Òò´Ë¾Í±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔ¶øÑÔ£¬Ö»ÐèÒª¿ØÖÆÕâ¸ö¿ØÖÆÌõ¼þ£¬Ï¡Êè¿ò¼ÜÏÂµÄLasso »Ø¹éÄ£ÐÍÍ¬Ñù¾ßÓÐ±äÁ¿Ñ¡ÔñÒ»ÖÂÐÔºÍÏàºÏÐÔ¡£

## ±¾ÕÂÐ¡½á

Õë¶ÔLasso»Ø¹éÄ£ÐÍµÄ¸Ä½øºÍÀ©Õ¹µÄ·½·¨ÓÐºÜ¶à£¬±¾ÎÄÒýÈëÁËÒ»ÖÖ¸üÎª¿í·ºµÄLasso »Ø¹éÄ£ÐÍ£¬¼´»ùÓÚÏ¡Êè¿ò¼ÜµÄLasso»Ø¹é£¬²¢ÔÚ´ËÖ®ÉÏÈÚÈëÁËÏÈÑéÐÅÏ¢µÄË¼Ïë¡£½ô½Ó×Å¸ø³öÁËÐÂÄ£ÐÍµÄ¾ßÌåÐÎÊ½£¬²¢ÇÒ¾ÍÁ½ÖÖ²»Í¬ÐÎÊ½µÄ ¸ø³öÁËÏàÓ¦µÄÇó½âËã·¨£¬×îºóËµÃ÷ÁËÐÂÄ£ÐÍËù¾ßÓÐµÄÁ¼ºÃÐÔÖÊ£¬ÎªºóÎÄµÄÊýÖµÊµÑé×ö³öÀíÂÛÖ§³Å¡£

# Ëæ»úÄ£ÄâÓëÊµÀýÕ¹Ê¾

ÎªÁËËµÃ÷»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso»Ø¹éÄ£ÐÍµÄÐÔÄÜÊÇÓÅÓÚÆÕÍ¨Lasso»Ø¹éÄ£ÐÍµÄ£¬ÔÚÎÒÃÇ½«´«Í³Lasso Ä£ÐÍºÍÐÂÄ£ÐÍÔÚÄ£ÄâºÍÊµÀýÊý¾ÝÉÏ½øÐÐÑÐ¾¿¡£¾ßÌåµØ£¬ÔÚÄ£ÄâÊµÑéÖÐÎÒÃÇ¼ÆËã²¢±È½Ï¸÷¸öÄ£ÐÍµÄ¾ù·½Îó²îºÍÔ¤²âÎó²î£¬²¢ÇÒ»­Í¼ÃèÊöÁË¸÷Ä£ÐÍ¼ÆËãµÄ²ÎÊýÖµ£¬ÔÚÊµÀý·ÖÎöÖÐÎÒÃÇ¼ÆËã²»Í¬Ä£ÐÍµÄÔ¤²âÎó²î¡£

## Êý¾ÝÄ£Äâ

ÎÒÃÇÔÚ¶à¸ö·ÂÕæÊý¾Ý¼¯ÉÏ½øÐÐÁËÊµÑé¡£¾ßÌåÊý¾ÝÓÉÒÔÏÂ·½Ê½Éú³É

Ã¿Ò»¸öÊý¾Ý¼¯¾ù°üº¬ÑµÁ·¼¯ºÍ²âÊÔ¼¯¡£Í³Ò»Éè¶¨²âÊÔ¼¯ÊýÁ¿ÎªÑù±¾ÊýÁ¿µÄ£¬Áî ÎªÉú³É µÄÏà¹ØÐÔ¾ØÕó£¬¼´ µÄÃ¿Ò»ÐÐ¶ÀÁ¢ÓÚ¡£ÎÒÃÇÊ¹ÓÃ À´±íÊ¾²ÎÊý µÄ¸öÊýºÍ¾ßÌåÊýÖµ¡£

²¢ÇÒÎªÁË¸üÐÎÏóµÄÃèÊöÏÈÑéÐÅÏ¢µÄ¶à¹Ñ£¬ÎÒÃÇ¶¨ÒåÒÑÖª²ÎÊý¹ØÏµ/×Ü²ÎÊý¸öÊý¡£¾Ù¸öÀý×Ó£¬, ¼ÙÉèÎÒÃÇÖªµÀ10¸öµÄ±ÈÀý¹ØÏµ£¬¼´ÔÚ50¸ö²ÎÊýÖÐ£¬ÒÑÖª10¸ö²ÎÊýµÄ¹ØÏµ£¬Ôò¼Ç¡£ ²¢ÇÒÔÚ½ÓÏÂÀ´µÄÊµÑéÖÐ£¬ÎÒÃÇ¾ù¼Ù¶¨ÒÑÖª²ÎÊý¼äµÄ¹ØÏµÎª1:1ÐÎÊ½£¬¼´ÔÚÉÏÃæÀý×ÓÖÐ£¬10¸ö ¾ùÏàµÈ¡£ ¾ßÌåÄ£ÐÍ²ÎÊýÈç±í[4.1](#biao4.1)ËùÊ¾£º

Ä£ÐÍ²ÎÊýÉèÖÃ

| Ä£ÐÍ | n | p |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 100 | 50 | 3 | 0.1 |  |
| 2 | 100 | 50 | 6 | 0.1 |  |
| 3 | 100 | 50 | 3 | 0.3 |  |
| 4 | 100 | 50 | 6 | 0.3 |  |
| 5 | 100 | 100 | 3 | 0.1 |  |
| 6 | 100 | 100 | 6 | 0.1 |  |
| 7 | 100 | 100 | 3 | 0.3 |  |
| 8 | 100 | 100 | 6 | 0.3 |  |
| 9 | 100 | 400 | 3 | 0.1 |  |
| 10 | 100 | 400 | 6 | 0.1 |  |
| 11 | 100 | 400 | 3 | 0.3 |  |
| 12 | 100 | 400 | 6 | 0.3 |  |

±í[4.1](#biao4.1)Õ¹Ê¾ÁËLassoÄ£ÐÍËùÓÐµÄ²ÎÊýÉèÖÃ¡£ÔÚ×Ô±äÁ¿ÊÇ·ñ¶ÀÁ¢µÄÇé¿öÏÂ£¬ÉèÖÃ²ÎÊýn ¾ùÎª100, ²ÎÊýp ·Ö±ðÎª50¡¢100¡¢400£¬·Ö±ð´ú±íµÍÎ¬Êý¾Ý¡¢³£¹æÊý¾Ý¡¢¸ßÎ¬Êý¾ÝÇé¿öÏÂµÄÄ£Äâ¡£ ÔÚ´¦ÀíÉÏÎÄËùÊ¾µÄÊý¾ÝÄ£ÄâÊ±£¬±¾ÎÄ¾ùÍ¨¹ýÓïÑÔÀ´ÊµÏÖÄ£ÐÍµÄ¹¹ÔìºÍÊý¾ÝµÄ´¦Àí(°üÀ¨µÄ±ê×¼»¯ºÍy µÄÖÐÐÄ»¯)¡£ÔÚÉÏÒ»½ÚÖÐÎÒÃÇ½éÉÜÁË¾ØÕó¾ßÓÐÁ½ÖÖ²»Í¬µÄÐÎÊ½£¬Òò´ËÔÚ¶ÔÉÏÊöÊý¾Ý½øÐÐÄ£ÄâÊ±£¬·Ö±ð¼ÇºÍ Îª»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜÏÂµÄLassoÄ£ÐÍ£¬µÄ¾ØÕóÐÎÊ½Îª¹«Ê½([[3.17]](#X759f390eb3fa9713deaf9cd3eab013a032432b6))£¬¼´Ï¡Êè¾ØÕó Îª·½Õó¡£ µÄ¾ØÕóÐÎÊ½Îª¹«Ê½ ([[3.18]](#X8145ffb8a66efa671aaa35835ea936b53c8492f))£¬¼´Ï¡Êè¾ØÕó Îª"ÊÝ"¾ØÕó¡£

¶ÔÓÚÉÏÊöÃ¿¸öÄ£ÐÍ¾ù½øÐÐÁË50´ÎÄ£Äâ£¬²¢Í¨¹ýÆ½¾ùµÄ¾ù·½Îó²îºÍÆ½¾ùµÄÔ¤²âÎó²î¶Ô¸÷·½·¨µÄÐÔÄÜ½øÐÐÁË²âÊÔ¡£

c@cccccc \*Ä£ÐÍ & & &  
& & & & & &   
  
1 & 0.250 & 21.850 & 0.194 & 19.299 & 0.231 & 21.097  
2 & 0.899 & 87.359 & 0.739 & 78.207 & 0.824 & 83.455  
3 & 0.248 & 22.745 & 0.082 & 13.428 & 0.189 & 19.711  
4 & 0.905 & 82.560 & 0.368 & 57.582 & 0.751 & 76.344  
  
1 & 0.297 & 18.077 & 0.201 & 15.357 & 0.257 & 17.132  
2 & 0.844 & 72.130 & 0.660 & 64.832 & 0.765 & 68.640  
3 & 0.261 & 18.055 & 0.061 & 12.373 & 0.196 & 16.331  
4 & 0.881 & 70.431 & 0.235 & 49.041 & 0.659 & 64.639

×¢£º²»¶ÀÁ¢ÇÒ·þ´Ó

±í[[biao4.2]](#biao4.2)Õ¹Ê¾ÁË50´ÎÄ£ÄâÖÐµÍÎ¬Êý¾Ý¸÷Ä£ÐÍµÄÆ½¾ù¾ù·½Îó²îºÍÆ½¾ùÔ¤²âÎó²î¡£´Ó×ÜÌåÀ´¿´£¬»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLassoÄ£ÐÍ¸÷ÐÔÄÜ¾ùÏÔÖøÓÅÓÚ´«Í³µÄLassoÄ£ÐÍ£¬²¢ÇÒ¾ÍÐÂÄ£ÐÍµÄÁ½ÖÖ¹¹Ôì·½·¨¶øÑÔ£¬Ä£ÐÍµÄÐÔÄÜÒªÓÅÓÚ Ä£ÐÍ£¬Õâ¿ÉÄÜÓëµÄÇó½âËã·¨ÓÐ¹Ø£¬ÒýÓÃ´«Í³µÄ×ø±êÏÂ½µ·¨À´Çó½â¸ÃÄ£ÐÍÎÞ·¨ÊÕÁ²µ½×îÓÅµã£¬ÕâµãÔÚShe(2010) µÄÎÄÕÂÖÐÒ²ÓÐ½øÐÐÏàÓ¦µÄ½âÊÍ¡£¶ÔÓÚ ÊÇ·ñ¶ÀÁ¢ÕâÒ»Ìõ¼þ¶øÑÔ£¬¶ÔÓÚÐÂÄ£ÐÍ¶øÑÔÓ°Ïì²»´ó£¬×ÜÌåµÄÐÔÄÜÓëÔ­Lasso Ä£ÐÍÍ¬µÈ½µµÍ¡£½Ó×ÅÎÒÃÇ¶Ô±È²ÎÊý µÄÇé¿ö£¬ÔÚµÍÎ¬Êý¾ÝÖÐ£¬µ±£¬ÎÞÂÛÊÇ»¹ÊÇÄ£ÐÍ£¬ÐÔÄÜÃ»ÓÐÔÚ Ê±ÌáÉýµÄÄÇÃ´ÏÔÖø¡£×îºó¶Ô±ÈÏÈÑé²ÎÊý µÄÇé¿ö£¬µ±ÒÑÖªÏÈÑéÐÅÏ¢Ôö¼ÓÊ±£¬ÐÂÄ£ÐÍµÄÐÔÄÜÌáÉýÐ§¹û·Ç³£Ã÷ÏÔ£¬²¢ÇÒÄ£ÐÍµÄÐÔÄÜÌáÉýÒªÏÔÖøÓÚÄ£ÐÍ¡£

Í¼[[tu4.1]](#tu4.1)ºÍÍ¼[[tu4.2]](#tu4.2)Õ¹Ê¾ÁËµÍÎ¬Êý¾ÝµÄ²ÎÊý¶Ô±È½á¹û¡£ºÚÉ«ÊµÏß´ú±íÕæÊµµÄ£¬ºìÉ« Ïß´ú±íÔ­LassoÄ£ÐÍµÄ²ÎÊý¹À¼Æ½á¹û£¬À¶É«Ïß´ú±í Ä£ÐÍµÄ²ÎÊý¹À¼Æ½á¹û£¬ÂÌÉ«Ïß´ú±íÄ£ÐÍµÄ²ÎÊý¹À¼Æ½á¹û¡£¶Ô±È·¢ÏÖ£¬ÔÚµÍÎ¬Êý¾ÝÖÐ£¬ Ä£ÐÍÄâºÏµÄ²ÎÊýÓëÕæÊµ×îÎª½Ó½ü£¬²¢ÇÒÌáÉý²¿·ÖÕýÊÇÎÒÃÇÒÑÖªµÄÏÈÑé²¿·Ö¡£ Ä£ÐÍÄâºÏµÄ²ÎÊýÂÔÓÅÓÚÔ­Lasso Ä£ÐÍ¡£½Ó×ÅÎÒÃÇÀ´¿´¿´ÔÚ³£¹æÊý¾ÝÖÐÊý¾ÝÄ£ÄâµÄ½á¹ûÈçºÎ£¬¾ßÌåÄ£Äâ½á¹ûÈçÏÂËùÊ¾£º

c@cccccc \*Ä£ÐÍ & & &  
& & & & & &   
  
5 & 1.714 & 189.242 & 1.05 & 127.938 & 1.462 & 169.264  
6 & 2.38 & 267.259 & 1.783 & 218.882 & 2.18 & 251.205  
7 & 1.556 & 165.9 & 0.132 & 23.274 & 1.084 & 120.331  
8 & 2.205 & 268.092 & 0.482 & 88.469 & 1.691& 217.199  
  
5 & 0.981 & 89.889 & 0.522 & 53.278 & 0.824 & 78.053  
6 & 1.686 & 172.062 & 1.245 & 131.391 & 1.607 & 162.259  
7 & 0.874 & 80.192 & 0.066 & 15.127 & 0.550 & 55.209  
8 & 1.675 & 169.902 & 0.255 & 62.936 & 1.092 & 127.909

×¢£º²»¶ÀÁ¢ÇÒ·þ´Ó

±í[[biao4.3]](#biao4.3)Õ¹Ê¾ÁË50´ÎÄ£ÄâÖÐ³£¹æÊý¾Ý¸÷Ä£ÐÍµÄÆ½¾ù¾ù·½Îó²îºÍÆ½¾ùÔ¤²âÎó²î£¬ÓëµÍÎ¬Êý¾ÝÀàËÆ£¬´Ó×ÜÌåÀ´¿´£¬»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLassoÄ£ÐÍ¸÷ÐÔÄÜÒ²ÏÔÖøÓÅÓÚ´«Í³µÄLasso Ä£ÐÍ£¬²¢ÇÒ¾ÍÐÂÄ£ÐÍµÄÁ½ÖÖ¹¹Ôì·½·¨¶øÑÔ£¬ Ä£ÐÍµÄÐÔÄÜÒªÓÅÓÚ Ä£ÐÍ¡£²»¹ýÏà½ÏÓÚµÍÎ¬Êý¾Ý¶øÑÔ£¬Ä£ÐÍµÄÐÔÄÜÌáÉý¸ü¼ÓÏÔÖø£¬ÈçÄ£ÐÍ7£¬,Æ½¾ù¾ù·½Îó²îËõÐ¡ÎªÔ­LassoÄ£ÐÍµÄ1/10¡£ ¶øÔÚµÍÎ¬Êý¾ÝÖÐ£¬ÌáÉýÐ§¹û×îºÃµÄÄ£ÐÍ3£¬Æ½¾ù¾ù·½Îó²îÒ²Ö»ËõÐ¡ÎªÔ­LassoÄ£ÐÍµÄ1/5¡£ ÀàËÆµÄ£¬ Ä£ÐÍÖÐÌáÉýµÄ×îÃ÷ÏÔËõÐ¡µ½Ô­Lasso Ä£ÐÍµÄ2/3, ¶øÔÚµÍÎ¬Êý¾ÝÖÐ£¬ÌáÉýÐ§¹û×îºÃµÄÄ£ÐÍ3£¬Æ½¾ù¾ù·½Îó²îÒ²Ö»ËõÐ¡ÎªÔ­LassoÄ£ÐÍµÄ8/9¡£

Í¼[[tu4.3]](#tu4.3)ºÍÍ¼[[tu4.4]](#tu4.4)Õ¹Ê¾ÁË³£¹æÊý¾ÝµÄ²ÎÊý¹À¼ÆÇé¿ö£¬Ïë½ÏÓÚµÍÎ¬Êý¾Ý£¬Í¼Æ¬ºÜºÃµÄÕ¹Ê¾ÁËÐÂÄ£ÐÍ²ÎÊý¹À¼ÆµÄÓÅÊÆ¡£ÔÚ Ïà»¥¶ÀÁ¢µÄÇé¿öÔÚ£¬ÔÚModel7 ÖÐ£¬ÎãÓ¹ÖÃÒÉ£¬ Ä£ÐÍÓÖÕ¹Ê¾ÁËÆäÁ¼ºÃµÄ²ÎÊý¹À¼Æ£¬²¢ÇÒ×îÈÃÈË¾ªÏ²µÄ£¬¾ÍºóÒ»ÖÖ·½·¨¹¹ÔìµÄ Ä£ÐÍ£¬ÂÌÉ«ÏßÏà±ÈÓÚºìÉ« Ïß£¬Ò²Òª¸üÌù½üÓëÕæÊµµÄ²ÎÊý£¬ÀàËÆµÄÇé¿öÔÚModel8 ÖÐÒ²ÓÐ±íÏÖ¡£

×îºó£¬±í[[biao4.4]](#biao4.4)Õ¹Ê¾ÁË50´ÎÄ£ÄâÖÐ¸ßÎ¬Êý¾Ý¸÷Ä£ÐÍµÄÆ½¾ù¾ù·½Îó²îºÍÆ½¾ùÔ¤²âÎó²î¡£ÎÞÂÛÊÇÔÚ²ÎÊýµÄÇé¿ö£¬Òà»òÕßÊÇÊÇ·ñ¶ÀÁ¢µÄÇé¿ö£¬ Ä£ÐÍÒÀÈ»Õ¹Ê¾ÁËÆäÁ¼ºÃµÄÐÔÄÜ£¬²¢ÇÒËæ×ÅÏÈÑéÐÅÏ¢ÁË½âµÄÔ½¶à£¬ÐÔÄÜµÄÌáÉý¾ÍÔ½Ã÷ÏÔ¡£ÕâÒ»µãÓëÇ°Á½ÖÖ×´¿ö²¢Ã»ÓÐÌ«´óÇø±ð¡£ºÜ¿ÉÏ§µÄÊÇ£¬Ä£ÐÍÔÚ¸ßÎ¬Êý¾ÝÇ°ÌáÏÂ£¬Æ½¾ù¾ù·½Îó²î ºÍÆ½¾ùÔ¤²âÎó²îÓëÔ­LassoÄ£ÐÍ²¢Ã»ÓÐ½Ï´ó²î±ð£¬ÉõÖÁÔÚÓÐÐ©Ä£ÐÍÖÐ£¬»¹Õ¹Ê¾³öÂÔÑ·ÓëÔ­Lasso Ä£ÐÍµÄÐÔÄÜ¡£²»¹ýÔÚÏÈÑéÐÅÏ¢ÁË½âÔö¼ÓµÄÇé¿öÏÂ£¬ÆäÆ½¾ù¾ù·½Îó²îºÍÆ½¾ùÔ¤²âÎó²îÒÀÈ»»¹ÊÇÓÅÓÚÔ­LassoÄ£ÐÍ¡£

c@cccccc \*Ä£ÐÍ & & &  
& & & & & &   
  
9 & 3.989 & 1706.42 & 3.701 & 1676.48 & 4.005 & 1681.42  
10 & 4.076 & 1951.67 & 3.785 & 1844.57 & 4.062 & 1907.11  
11 & 3.995 & 1760.58 & 1.647 & 827.79 & 4.071 & 1686.69  
12 & 4.015 & 1746.83 & 1.660 & 826.386 & 4.034 & 1694.45  
  
9 & 4.711 & 3235.7 & 4.206 & 2925.77 & 4.753 & 3232.29  
10 & 4.565 & 3356.81 & 4.177 & 2921.72 & 4.494 & 3281.5  
11 & 4.575 & 3444.8 & 1.239 & 733.656 & 4.755 & 3189.480  
12 & 4.755 &3338.38 & 1.218 & 691.715 & 4.725 & 3187.44

×¢£º²»¶ÀÁ¢ÇÒ·þ´Ó

ÓëÇ°Á½ÖÖÊý¾Ý×´¿öÀàËÆ£¬Í¼[[tu4.5]](#tu4.5)ºÍÍ¼[[tu4.6]](#tu4.6)Õ¹Ê¾ÁË¸ßÎ¬Êý¾ÝµÄ²ÎÊý¹À¼ÆÇé¿ö¡£ Ä£ÐÍÒÀÈ»±íÏÖÁËÆäÁ¼ºÃµÄ²ÎÊý¹À¼Æ×´¿ö£¬ÎÞÂÛÊÇÏÈÑéÐÅÏ¢ÒÑÖªµÄ¶à¹Ñ£¬ÔÚÍ¼Æ¬ÖÐ¾ùºÜºÃµÄ±íÏÖÁË³öÀ´¡£²¢ÇÒÓëÇ°Ãæ·ÖÎöµÄÒ»ÖÂ£¬ Ä£ÐÍÓëÔ­LassoÄ£ÐÍËäÈ»ÔÚ±í[[biao4.4]](#biao4.4)ÖÐ¿ÉÒÔÖ±¹ÛµÄ¿´³öÇø±ð£¬µ«ÔÚÍ¼Æ¬ÖÐ¶þÕß¼¸ºõÒ»ÖÂ¡£

## ÊµÖ¤·ÖÎö

### Ñù±¾Êý¾Ý¸Å¿ö

ÎªÁË¸üºÃµÄËµÃ÷±¾ÎÄÌá³öµÄ»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLassoÄ£ÐÍÐÔÄÜÊÇÓÅÓÚÔ­Lasso Ä£ÐÍ£¬ÎÒÃÇ½«ÔÚ±¾½ÚÖÐ²ÉÓÃÕæÊµÊý¾Ý¼¯¶ÔÆä½øÐÐÊµÑéÑéÖ¤¡£´Ó´óÊý¾Ý¾ºÈüÆ½Ì¨ ÖÐ»ñÈ¡ÁËNBA ÇòÔ±Êý¾Ý(https://www.kaggle.com/drgilermo/nba-players-stats)¡£

¸ÃÊý¾Ý¼¯°üº¬67¸öNBAÈü¼¾µÄ¸öÈËÍ³¼ÆÊý¾Ý£¬Êý¾ÝÐÅÏ¢Èç±í[4.2](#biao4.5)ËùÊ¾£º

Êý¾ÝÐÅÏ¢ËµÃ÷

| ÎÄ¼þ | Ñù±¾Á¿ | ÌØÕ÷Á¿ | ËµÃ÷ |
| --- | --- | --- | --- |
|  | 4551 | 8 | ¼ÇÂ¼ÇòÔ±¸öÈËÐÅÏ¢ |
|  | 3923 | 7 | ¼ÇÂ¼ÇòÔ±¸öÈËÐÅÏ¢ |
|  | 24.7k | 53 | ¼ÇÂ¼ÇòÔ±Ð§Á¦NBAÆÚ¼äµÄ¸÷ÏîÊý¾Ý |

ÆäÖÐÎÄ¼þ¼ÇÂ¼ÁË¸÷ÇòÔ±ÐÅÏ¢£¬°üÀ¨£ºÐÕÃû¡¢Ð§Á¦¿ªÊ¼Ê±¼ä¡¢Ð§Á¦½ØÖ¹Ê±¼ä¡¢Î»ÖÃ¡¢´óÑ§Ãû³Æ¡¢Éí¸ß¡¢ÌåÖØ¡¢³öÉúÈÕÆÚ8¸öÁÐÌØÕ÷¡£

ÎÄ¼þ¼ÇÂ¼¸÷ÇòÔ±ÀàËÆµÄÐÅÏ¢£¬°üÀ¨£ºÐÕÃû¡¢³öÉúÊ¡·Ý¡¢³öÉúµØÇø¡¢Éí¸ß¡¢ÌåÖØ¡¢´óÑ§¡¢³öÉúÈÕÆÚ7 ¸öÁÐÌØÕ÷¡£

×îÖØÒªµÄÊý¾Ý¼ÇÂ¼ÁË¸÷ÇòÔ±ÔÚÐ§Á¦NBAÆÚ¼äµÄ¸÷ÏîÊý¾Ý£¬°üÀ¨ÉÏ³¡Ê±¼ä¡¢Öú¹¥¡¢ÃüÖÐÂÊ¡¢ÄêÁä¡¢µÃ·Ö¡¢Ð§Á¦¶ÓÎéµÈ53¸öÁÐÌØÕ÷¡£

ÓÉÓÚÎ´ÖªÔ­Òò£¬¸ÃÊý¾Ý¼¯ÔÚ1950-1973 Äê¼ä²¢Ã»ÓÐÍ³¼Æ½ø¹¥Àº°åÂÊ¡¢·ÀÊØÀº°åÂÊ¡¢×ÜÀº°åÂÊ¡¢³ö³¡ÂÊ¡¢ÇÀ¶ÏÂÊµÈ13¸öÌØÕ÷±äÁ¿¡£µ«ÔÚ1974-1980Äê¼äÕâÐ©ÌØÕ÷¾ùÓÐÏàÓ¦µÄ¼ÇÂ¼¡£¶ø±¾ÊµÑéµÄÄ¿±êÊÇÎªÁË·ÖÎöÔÚÁ½·ÖÇòÊ±´ú(1980ÄêÒÔÇ°)£¬¸÷ÏîÌØÕ÷¶ÔÇòÔ±¹±Ï×ÂÊµÄÓ°Ïì¡£ÎªÁË³ä·ÖÀûÓÃÍêÕûÊý¾Ý¼¯£¬±¾ÎÄ½«ÀûÓÃ1950-1973 ÄêµÄÊý¾Ý·Ö±ð×öLasso »Ø¹éÄâºÏºÍ×îÐ¡¶þ³ËÄâºÏ£¬½«¹Û²ìµÃ³öµÄÌØÕ÷¼äµÄ±ÈÀý¹ØÏµ×÷ÎªÏÈÑéÐÅÏ¢¡£ÔÚ´ËÖ®ºóÀûÓÃ1973-1980ÄêµÄÊý¾Ý£¬·Ö±ð»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso Ä£ÐÍºÍÔ­LassoÄ£ÐÍ×öÏàÓ¦µÄ»Ø¹éÄâºÏ£¬¶Ô±È²»Í¬Ä£ÐÍ²»Í¬ÏÈÑéÇé¿öÏÂµÄÐÔÄÜ²î¾à¡£

### Êý¾Ý´¦Àí¼°ÊµÑé½á¹û

ÔÚ½øÐÐÊý¾Ý·ÖÎöºÍ½¨Ä£µÄ¹ý³ÌÖÐ£¬Êý¾Ý´¦ÀíÊÇ±Ø²»¿ÉÉÙµÄ²½Öè£¬ÒòÎªÍùÍùÔ­Ê¼Êý¾Ý²¢²»ÄÜÍêÈ«Âú×ã¹¹ÔìÄ£ÐÍµÄÒªÇó¡£Í¬ÑùµÄ£¬¶ÔÓÚ±¾ÎÄÊ¹ÓÃµÄNBAÊý¾Ý£¬¼´Ê¹ÆäÔ­×÷ÕßÒÑ¾­½øÐÐÁË²¿·ÖÊÊµ±µÄ´¦Àí£¬µ«ÎªÁËÊÊÓÃ±¾ÎÄµÄÄ¿±êÒªÇó£¬ÎÒÃÇÈÔÐèÒª½«Êý¾Ý½øÐÐÏàÓ¦µÄ´¦Àí¡£ÔÚ´ËÖØÉê£¬±¾ÎÄÔÚ´¦ÀíÉÏÎÄËùÊ¾µÄÊý¾Ý¼¯Ê±£¬¾ùÍ¨¹ýÀ´ÊµÏÖÄ£ÐÍµÄ¹¹ÔìºÍÊý¾ÝµÄ´¦Àí¡£Êý¾Ý´¦Àí¾ßÌå²½ÖèÈçÏÂ£º

1. ½«Èý¸öÎÄ¼þ°´ÕÕÇòÔ±ÐÕÃûÎª¹Ø¼ü´ÊºÏ²¢£¬Ñ¡³ö1950-1980Äê¼äµÄÇòÔ±ÐÅÏ¢¡£
2. ½«ºÏ²¢ºóµÄÎÄ¼þ°´ÕÕÇòÔ±ÐÕÃû·Ö×é£¬ÒÔ¹±Ï×ÂÊ¸ßµÍ£¬Ñ¡³öÃ¿¸öÇòÔ±Ð§Á¦NBA ÆÚ¼äµÄ×î¼Ñ¹±Ï×ÂÊ¡£
3. ÌÞ³ýÈ±Ê§Öµ½Ï¶àµÄ±äÁ¿£¬°üÀ¨³öÉúÈÕÆÚ¡¢Ê¡·ÝºÍµØÇø¡£´´ÔìÐÂ±äÁ¿¡°Ð§Á¦Ê±³¤¡±£ºÐ§Á¦Ê±³¤= Ð§Á¦½ØÖ¹Ê±¼ä-Ð§Á¦¿ªÊ¼Ê±¼ä¡£
4. ¶ÔÓÚÉÙÊý¾ßÓÐÈ±Ê§ÖµµÄ±äÁ¿£¬ÒÔ¸ÃÌØÕ÷¾ùÖµ½øÐÐÌî³ä¡£¶ÔÓÚ¶¨ÀàµÄÌØÕ÷±äÁ¿£¬±¾ÎÄ´¦ÀíµÄ·½Ê½ÊÇ½«Æä×ª»¯ÎªÑÆ±äÁ¿¡£×îºó£¬ÎÒÃÇ»á½«ËùµÃµ½µÄÊý¾Ý¾ØÕó½øÐÐ±ê×¼»¯´¦Àí¡£

Í¨¹ýÉÏÊöÊý¾Ý´¦Àíºó£¬ÎÒÃÇµÃµ½±¾ÎÄÐèÒªµÄÑù±¾Êý¾Ý¼¯Îª¡£ ÆäÖÐ1950-1973 ÄêµÄÊý¾Ý¼¯Îª£¬½«Æä¼ÇÎªÏÈÑéÊý¾Ý¼¯£»1974-1980ÄêµÄÊý¾Ý¼¯Îª£¬½«Æä¼ÇÎªÊµÖ¤Êý¾Ý¼¯¡£²¢ÇÒÉÏÊöÊý¾Ý¼¯ÖÐ¾ùÓÐ255 ¸öÎªÑÆ±äÁ¿¡£

ÓÉÓÚÔÚÄ£ÄâÊµÑéÖÐÐÎÊ½±íÏÖµÄÐÔÄÜÒªÆÕ±éÓÅÓÚ ÐÎÊ½£¬Òò´ËÔÚÊµÖ¤·ÖÎöÖÐ£¬±¾ÎÄÖ»Ñ¡ÓÃÐÎÊ½µÄÐÂÄ£ÐÍ½øÐÐÊµÖ¤¡£²¢ÇÒÇ°ÎÄÖÐ½éÉÜµÄÏÈÑéÐÅÏ¢ÊÇÓÉ×îÐ¡¶þ³Ë¹À¼ÆºÍLasso¹À¼ÆÁ½ÖÖ¹À¼Æ·½·¨µÃµ½µÄ²ÎÊý±ÈÀý£¬Òò´ËÎÒÃÇ½«Á½ÖÖµÃ³öÏÈÑéµÄÐÎÊ½·Ö±ð¼ÇÎª ºÍ£¬Ç°ÕßÎªÀûÓÃ×îÐ¡¶þ³ËµÃ³öµÄÏÈÑé£¬ºóÕßÎªÀûÓÃLasso µÃ³öµÄÏÈÑé¡£

ÎÒÃÇÈ¡³öÏÈÑéÊý¾Ý¼¯£¬½«¹±Ï×ÂÊ×÷ÎªÏìÓ¦±äÁ¿¾ØÕó£¬È¥³ýÄê·ÝºÍÇòÔ±ÐÕÃûºóµÄÊý¾Ý¼¯×÷Îª½âÊÍ±äÁ¿¾ØÕó¡£ ÀûÓÃ×îÐ¡¶þ³Ë¹À¼Æ½øÐÐ50´ÎÊý¾ÝÄâºÏ¡£¹Û²ìÄâºÏ²ÎÊý¼äµÄ±ÈÀý¹ØÏµ¡£

×îÐ¡¶þ³Ë²ÎÊý¹À¼Æ½á¹ûÕ¹Ê¾

| ÌØÕ÷ | mean | std | median |
| --- | --- | --- | --- |
| ·¸¹æ´ÎÊý | -0.003 | 0.001 | 0.003 |
| ÉÏ³¡Ê±¼ä | 0.002 |  | 0.002 |
| Éí¸ß | 0.004 | 0.008 | 0.004 |
| ÌåÖØ | -0.006 | 0.005 | -0.006 |
| Ð§Á¦Ê±³¤ | 0.057 |  | 0.055 |
| ÄêÁä | -0.011 | 0.001 | -0.012 |
| ·£ÇòÃüÖÐ´ÎÊý | -0.011 | 0.002 | 0.011 |
| ·£ÇòÃüÖÐÂÊ | 0.001 |  |  |

±í[4.3](#biao4.6)Õ¹Ê¾ÁË×îÐ¡¶þ³Ë¹À¼ÆÔÚ50´ÎÄ£ÄâÖÐ²ÎÊý¹À¼ÆÁ¿µÄÃèÊöÐÔÍ³¼Æ£¬ÆäÖÐmean¡¢std¡¢median ·Ö±ð´ú±í50´ÎÄ£ÄâÖÐ²ÎÊý¹À¼ÆÖµµÄ¾ùÖµ¡¢±ê×¼²îºÍÖÐÎ»Êý¡£ÓÉÓÚ¿¼ÂÇ²ÎÊýÌ«Ð¡ÔÚÀûÓÃÐÂÄ£ÐÍ½øÐÐÄâºÏÊ±»áÑ¹ËõÖÁ0£¬±¾ÎÄÑ¡È¡¾ùÖµ, ÇÒ±ê×¼²î½ÏÐ¡µÄ²ÎÊý¹À¼ÆÁ¿¡£ÆäÖÐ°üÀ¨£º·¸¹æ´ÎÊý¡¢ÉÏ³¡Ê±¼ä¡¢Éí¸ß¡¢ÌåÖØ¡¢Ð§Á¦Ê±³¤¡¢ÄêÁä¡¢·£ÇòÃüÖÐ´ÎÊý¡¢·£ÇòÃüÖÐÂÊ8¸öÌØÕ÷±äÁ¿¡£´ÓÉÏÊöÈý¸öÖ¸±ê¿´£¬Ã¿¸öÌØÕ÷¾ù¾ßÓÐÁ¼ºÃµÄ´ú±íÐÔ¡£Í¬Ê±ÎªÁË¸ü¼òµ¥µÄ±íÊ¾¾ØÕó£¬·Ö±ð½«ÉÏÊö8 ¸öÌØÕ÷±íÊ¾Îª£¬²¢²ÉÈ¡Á½Á½À¦°óµÄÐÎÊ½£¬¼´,,,¡£ Òò´Ë£¬×îÐ¡¶þ³Ë¹À¼ÆÐÎ³É ¾ØÕó°üÀ¨¡£

ÁË½âÍê×îÐ¡¶þ³Ë¹À¼ÆÄâºÏµÄ²ÎÊý£¬ÎÒÃÇ¼ÌÐø¹Û²ìLasso¹À¼ÆÄâºÏµÄ²ÎÊý£¬Í¬×îÐ¡¶þ³Ë¹À¼ÆÀàËÆ£¬ÎÒÃÇÈ¡³öÏÈÑéÊý¾Ý¼¯£¬½«¹±Ï×ÂÊ×÷ÎªÏìÓ¦±äÁ¿¾ØÕó£¬È¥³ýÄê·ÝºÍÇòÔ±ÐÕÃûºóµÄÊý¾Ý¼¯×÷Îª½âÊÍ±äÁ¿¾ØÕó¡£ÀûÓÃLasso¹À¼Æ½øÐÐ50´ÎÊý¾ÝÄâºÏ¡£¹Û²ìÄâºÏ²ÎÊý¼äµÄ±ÈÀý¹ØÏµ¡£

Lasso¹À¼Æ½á¹ûÕ¹Ê¾

| ÌØÕ÷ | mean | std | median |
| --- | --- | --- | --- |
| Í¶ÖÐ´ÎÊý | -0.016 |  | -0.016 |
| ·¸¹æ´ÎÊý | 0.014 |  | 0.014 |
| ÉÏ³¡Ê±¼ä | 0.002 |  | 0.002 |
| µÃ·ÖÃüÖÐÂÊ | 0.003 | 0.001 | 0.003 |
| Öú¹¥ | -0.002 |  | 0.002 |

±í[4.4](#biao4.7)Õ¹Ê¾ÁËLasso¹À¼ÆÔÚ50´ÎÄ£ÄâÖÐ²ÎÊý¹À¼ÆÁ¿µÄÃèÊöÐÔÍ³¼Æ£¬ÆäÖÐmean¡¢std¡¢median ·Ö±ð´ú±í50 ´ÎÄ£ÄâÖÐ²ÎÊý¹À¼ÆÖµµÄ¾ùÖµ¡¢±ê×¼²îºÍÖÐÎ»Êý¡£²ÉÈ¡Óë×îÐ¡¶þ³ËÏàÍ¬µÄ±ê×¼Ñ¡È¡²ÎÊý¹À¼ÆÁ¿£¬ÆäÖÐ°üÀ¨£ºÍ¶ÖÐ´ÎÊý¡¢·¸¹æ´ÎÊý¡¢ÉÏ³¡Ê±¼ä¡¢µÃ·ÖÃüÖÐÂÊ¡¢Öú¹¥5¸öÌØÕ÷±äÁ¿¡£Óë×îÐ¡¶þ³Ë¹À¼Æ½á¹ûÀàËÆ£¬5 ¸öÌØÕ÷±äÁ¿¾ù¾ßÓÐÁ¼ºÃµÄ´ú±íÐÔ¡£Í¬Ñù£¬½«ÉÏÊö5 ¸öÌØÕ÷±äÁ¿±íÊ¾Îª£¬ÓÉÓÚLasso¹À¼ÆµÃµ½µÄÌØÕ÷±äÁ¿½ÏÉÙ£¬Òò´Ë±¾ÎÄ½«Ç°Á½¸ö±äÁ¿À¦°ó£¬ºóÈý¸ö±äÁ¿À¦°ó¡£¼´,¡£Òò´Ë£¬Lasso¹À¼ÆÐÎ³ÉµÄ¾ØÕó°üÀ¨ ¡£

ÔÚ´¦ÀíÍêÏÈÑéÊý¾Ý¼¯µÃ³öÏÈÑéÐÅÏ¢ºó£¬ ÎÒÃÇÈ¡³öÊµÖ¤Êý¾Ý¼¯£¬¸ÃÊý¾Ý¼¯Ïà±ÈÇ°ÕßÔö¼ÓÁË13 ¸öÌØÕ÷±äÁ¿¡£±¾ÎÄÔËÓÃÏÈÑéÊý¾Ý¼¯¹À¼ÆµÄ²ÎÊý±ÈÀý¹ØÏµ×÷ÎªÏÈÑéÐÅÏ¢£¬ÀûÓÃÉÏ½ÚÖÐÌá³öµÄÄ£ÐÍ£¬·Ö±ð½«×îÐ¡¶þ³ËºÍLasso¹À¼ÆµÃ³öµÄÏÈÑé²ÎÊý±ÈÀý¹ØÏµ£¬ÈÚÈë Ä£ÐÍÖÐ£¬·Ö±ðÈ¡ÃûÎªºÍ¡£

c@cccccc \*Ä£ÐÍ & &  
& mean & std & median & mean & std & median  
 & 0.469 & 0.075 & 0.469 & 0.473 & 0.043 & 0.475  
 & 0.459 & 0.068 & 0.448 & 0.464 & 0.037 & 0.462  
 & 0.455 & 0.068 & 0.450 & 0.461 & 0.038 & 0.458

±í[[biao4.8]](#biao4.8)Õ¹Ê¾ÁËÁ½ÖÖÏÈÑéÐÅÏ¢Çé¿öÏÂÐÂÄ£ÐÍÓëÔ­LassoÄ£ÐÍµÄÐÔÄÜ±È½Ï¡£ÓÉÓÚÕæÊµ²ÎÊýÎ´Öª£¬ÎªÁË¸üºÃµÄÌåÏÖÐÂÄ£ÐÍµÄ×÷ÓÃ£¬±¾ÎÄÔÚÊµÖ¤²¿·ÖÔö¼ÓÁËÒ»¸öÆÀ¼ÛÖ¸±ê£¬¼´Æ½¾ù¾ø¶ÔÎó²î(Mean Absolute Error )£º,¡£ÆäÖÐ ÎªÏìÓ¦±äÁ¿£¬ Îª¸ù¾ÝÄ£ÐÍÔ¤²âµÄÏìÓ¦±äÁ¿¡£Í¬Ê±ÎªÁËºÏÀíµÄ±íÊ¾ÕæÊµÊý¾ÝµÄÐÔÄÜ£¬ÎÒÃÇ¶ÔÊµÖ¤Êý¾Ý¼¯×öÁË50 ´ÎÊÔÑé£¬ÉÏ±íÖÐmean¡¢std¡¢median·Ö±í±íÊ¾ ºÍ50´ÎÊÔÑé½á¹ûµÄ¾ùÖµ¡¢±ê×¼²îºÍÖÐÎ»Êý¡£

´Ó±í[[biao4.8]](#biao4.8)Õ¹Ê¾µÄ½á¹û¿ÉÖª£¬»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLassoÄ£ÐÍ´Ó¾ùÖµºÍÖÐÎ»ÊýÉÏ¿´×ÜÌåÊÇÓÅÓÚÔ­LassoÄ£ÐÍµÄ£¬²¢ÇÒÎÈ¶¨ÐÔÒ²ÂÔÓÅÓÚÔ­LassoÄ£ÐÍ¡£ÔÚÀûÓÃLASSO ¹À¼ÆµÃ³öµÄÏÈÑé²ÎÊý±ÈÀýÊ±£¬ÎÞÂÛÊÇ ÉÏºÍ ÉÏ¾ùµÃµ½ÁËÏÔÖøµÄÌáÉý£¬ÓÈÆäÊÇÔÚÏÈÑéÐÅÏ¢ºÜÉÙµÄÇé¿öÏÂ£¬ÎÒÃÇÖ»ÁË½âÁË5¸öÌØÕ÷µÄ¹ØÏµ£¬»»Ëã³É µÄÇé¿öÏÂ£¬ ÌáÉýÁË£¬ÌáÉýÁË¡£ÔÚÀûÓÃLasso ¹À¼ÆµÃµ½µÄÏÈÑé²ÎÊý±ÈÀý£¬Óë×îÐ¡¶þ³Ë¹À¼ÆÀàËÆ£¬ÎÒÃÇ½«ÏÈÑéÐÅÏ¢½øÐÐ»»Ëã£¬ µÄÇé¿öÏÂ£¬ ÌáÉýÁË£¬ÌáÉýÁË¡£

## ±¾ÕÂÐ¡½á

Í¨¹ýÄ£ÄâºÍÊµÖ¤µÄ½á¹ûÀ´¿´£¬ÎÒÃÇµÃ³öÁËÒÔÏÂ½áÂÛ£º

1. »ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso Ä£ÐÍ×ÜÌåÐÔÄÜÊÇÓÅÓÚÔ­Ê¼LassoÄ£ÐÍ£¬ÎÞÂÛÏÈÑéÏ¡Êè¾ØÕóÊÇ·½Õó»¹ÊÇ"ÊÝ"¾ØÕó£¬Òò´ËÀûÓÃÏÈÑéÏ¡Êè¿ò¼ÜµÄ·½·¨¾ßÓÐÁ¼ºÃµÄ¿ÉÐÐÐÔºÍÒ»¶¨µÄÍÆ¹ãÐÔ¡£
2. ¶ÔÓÚÄ³ÖÖÔ­Òòµ¼ÖÂ²¿·ÖÑù±¾µÄ²¿·ÖÌØÕ÷´æÔÚÈ±Ê§Ê±£¬¿ÉÒÔÀûÓÃ»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLassoÄ£ÐÍ½øÐÐ´¦Àí£¬»áµÃµ½ÓÅÓÚÔ­LassoÄ£ÐÍµÄ½á¹û¡£Òò´ËÊ¹ÓÃ¸ÃÄ£ÐÍ´æÔÚÒ»¶¨µÄ¾ÖÏÞÐÔ¡£
3. ÔÚÊý¾Ý´¦ÀíÖÐ£¬Èç¹ûÏÈÑéÐÅÏ¢»ñÈ¡µÄ±ÈÀýÔ½¸ß£¬»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso Ä£ÐÍµÄÐÔÄÜÏà±ÈÔ­Ê¼LassoÄ£ÐÍ¾ÍÔ½ºÃ¡£

# ×Ü½áÓëÕ¹Íû

## ÑÐ¾¿½áÂÛ

ÔÚÊý¾ÝÁ¿³ÊÏÖ±¬Õ¨Ê½Ôö³¤µÄ½ñÌì£¬Ó¿ÏÖ³öÁË¸÷Ê½¸÷ÑùµÄÊý¾ÝÄ£ÐÍ¡£Óë´ËÍ¬Ê±£¬±äÁ¿Ñ¡ÔñÎÊÌâÒ»Ö±ÊÇÍ³¼ÆÑ§ÀíÂÛºÍÓ¦ÓÃÖÐµÄÒ»¸öÖØÒªÎÊÌâ£¬Òò´ËÈçºÎÑ¡ÔñÒ»ÖÖºÃµÄ±äÁ¿Ñ¡Ôñ·½·¨ÊÇºÜÓÐ±ØÒªµÄ¡£ºÃµÄ±äÁ¿Ñ¡Ôñ²»½ö¿ÉÒÔ±ÜÃâÎ¬¶ÈÔÖÄÑµÄÎÊÌâ£¬»¹¿ÉÒÔ±£Ö¤Ô¤²âµÄ¾«×¼¶È¼°Ä£ÐÍµÄÎÈ¶¨ÐÔ¡£ÆäÖÐÊ¹ÓÃ×î¶àµÄÄ£ÐÍ¾ÍÊÇLasso Ä£ÐÍ£¬Ïà½ÏÓÚ´«Í³µÄ»Ø¹éÄ£ÐÍ£¬ÆäÖ÷ÒªµÄÓÅÊÆÔÚÓÚ¿ÉÒÔ¶Ô¶àÓàµÄ±äÁ¿½øÐÐÑ¹Ëõ²¢ÌÞ³ýÓ°Ïì³Ì¶È½ÏµÍµÄ±äÁ¿£¬´Ó¶øºÜºÃµÄ´ïµ½±äÁ¿Ñ¡ÔñµÄ×÷ÓÃ¡£Òò´Ë£¬ÔÚ´¦Àí¸ßÎ¬Êý¾ÝÇé¿öÖÐ£¬LassoÄ£ÐÍ¾ßÓÐºÜºÃµÄÊÊÓÃÐÔ£¬²¢ÇÒÆä¸Ä½ø·½·¨ÔÚÍ³¼ÆÑ§ÑÐ¾¿ÁìÓòÖÐÒ²Öð½¥ÊÜµ½ÈËÃÇµÄ¹Ø×¢¡£

±¾ÎÄÔÚ²éÔÄÁË´óÁ¿µÄ¹úÄÚÍâÏà¹ØÎÄÏ×µÄ»ù´¡ÉÏ£¬Ìá³öÁË»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜÏÂµÄLasso »Ø¹éÄ£ÐÍµÄÑÐ¾¿¼°ÆäÊµ¼ÊÔËÓÃ£¬ÂÛÎÄµÄÖ÷Òª¹¤×÷×Ü½áÈçÏÂ£º

1. LassoÄ£ÐÍ×÷Îª±äÁ¿Ñ¡ÔñÁìÓòµÄÖØÒªÄ£ÐÍ£¬ÔÚ±£Áô¸ß¾«×¼¶ÈµÄÇ°ÌáÏÂÆðµ½ÁË±äÁ¿Ñ¡ÔñµÄ×÷ÓÃ£¬¿ÉÒÔÓÐÐ§µÄ½â¾öÎ¬¶ÈÔÖÄÑµÄÎÊÌâ£¬Òò´ËÔÚ±¾ÎÄµÄ¿ªÆªÉîÈëÇ³³öµÄ²ûÊöÁËLasso »Ø¹éÄ£ÐÍµÄÔ­Àí¡¢Çó½âËã·¨¼°ÆäÐÔÖÊ£¬Ö¤Ã÷ÁËÆäÔÚ±äÁ¿Ñ¡ÔñµÄÓÅÊÆºÍÌØµã¡£
2. ÏÈÑéÐÅÏ¢×÷Îª×ÔÓÉÊôÐÔµÄÒ»²¿·Ö£¬Æä±¾Éí¾ÍÊÇ´æÔÚµÄ£¬±¾ÎÄÌá³öÁËÒ»ÖÖ½«ÏÈÑéÐÅÏ¢ÈÚÈëLassoÄ£ÐÍµÄ·½·¨£¬½«Æä³ÆÎª»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜÏÂµÄLasso »Ø¹éÄ£ÐÍ¡£½ô½Ó×Å¾ÍÈÚºÏ·½Ê½Ìá³öÁËÁ½ÖÖ²»Í¬µÄÐÎÊ½£¬²¢¾Í²»Í¬ÐÎÊ½·Ö±ð¸ø³öÁËÏàÓ¦µÄÇó½âËã·¨£¬²¢¶ÔÐÂÄ£ÐÍµÄÐÔÖÊ½øÐÐÁËÖ¤Ã÷¡£
3. ÎÄÕÂµÄ×îºó¾ÍÐÂÄ£ÐÍµÄ²»Í¬ÐÎÊ½·Ö±ð½øÐÐÁËÊý¾ÝÄ£ÄâºÍÊµÖ¤·ÖÎö£¬ÎÞÂÛÊÇ¸ßÎ³¡¢µÍÎ¬»¹ÊÇ³£¹æÊý¾ÝÇé¿öÏÂ£¬Æä½á¹û±íÃ÷»ùÓÚÏÈÑéÏ¡Êè¿ò¼ÜµÄLasso Ä£ÐÍ×ÜÌå±íÏÖÊÇÓÅÓÚ´«Í³Lasso Ä£ÐÍµÄ¡£²¢ÇÒ¾ÍÈÚºÏÐÎÊ½¶øÑÔ£¬¾ØÕóÎª·½ÕóÊ±Ð§¹ûÓÅÓÚ ¾ØÕóÎª"ÊÝ"¾ØÕóµÄÇé¿ö¡£

## ±¾ÎÄÕ¹Íû

±¾ÎÄÔÚ½áºÏÎÄÏ×ºÍÑÐ¾¿ÄÚÈÝºó£¬Ìá³öÒÔÏÂÐèÒªÑÐ¾¿µÄ·½Ïò£º

1. LassoÄ£ÐÍÌØ±ðÊÊÓÃÓÚ¸ßÎ¬Êý¾ÝÇé¿öÏÂ£¬¶ø»ùÒòÊý¾ÝºÜ·ûºÏÕâÖÖÊý¾Ý½á¹¹£¬²¢ÇÒ¾Í»ùÒòÑ§ÖÐ´æÔÚ»ùÒòÍ¨Â·¡¢ÖØµþ»ùÒò¡¢µÈÎ»»ùÒòµÈ¾ßÓÐÏÈÑéÐÅÏ¢µÄËµ·¨¡£Òò´Ë¿ÉÒÔÊÕ¼¯·ûºÏ±¾ÎÄÒªÇóµÄÊý¾Ý½øÐÐ¸ü¶àµÄÊµÖ¤·ÖÎö£¬¸ø»ùÒòÑ§Ìá¹©½è¼øÒâÒå¡£
2. ÊµÑéÊý¾ÝÐèÒªÏÈÑéÐÅÏ¢£¬²¢ÇÒ¸ÃÏÈÑéÐÅÏ¢ÄÜ¹»ÎªÎÒËùÓÃ£¬Òò´Ë¸Ã·½·¨¾ßÓÐÒ»¶¨µÄ¾ÖÏÞÐÔ£¬²¢²»ÊÇËùÓÐÊý¾Ý¾ùÄÜÓÃ¸Ã·½·¨½øÐÐÊµÑé¡£ÈçºÎÓÐÐ§µÄ´òÆÆ±¾ÎÄÊý¾ÝµÄ¾ÖÏÞÐÔ£¬ÈÔÊÇ½ÓÏÂÀ´ÐèÒª½â¾öµÄ·½Ïò¡£
3. ½«ÏÈÑéÐÅÏ¢ÈÚÈëÄ£ÐÍµÄË¼Ïë²»½ö½ö´æÔÚÓÚLasso»Ø¹éÄ£ÐÍ£¬¸ÃÏë·¨¿ÉÒÔÈÚÈë¹ã·ºµÄÊý¾ÝÄ£ÐÍ£¬Ö»Òª¸ÃÄ£ÐÍ¾ßÓÐÕýÔò»¯µÄ½á¹¹£¬¾ù¿ÉÒÔ½«±¾ÎÄµÄ·½·¨½øÐÐÈÚºÏ¡£
4. ´ÓÊµÖ¤·ÖÎöÀ´¿´£¬Èç¹ûÎÒÃÇ»ñµÃ¾ßÓÐÏàËÆ½á¹¹µÄÊý¾Ý£¬¼´ÓÉÓÚÖÖÖÖÔ­Òòµ¼ÖÂ²¿·ÖÌØÕ÷Êý¾Ý´óÁ¿È±Ê§£¬¾ù¿ÉÒÔÀûÓÃ±¾ÎÄÌá³öµÄÐÂÄ£ÐÍ½øÐÐ´¦Àí£¬¸Ã·½·¨¸øÓëÁËÊý¾ÝÇåÏ´Ò»ÖÖÐÂË¼Â·¡£

1.1

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ÔÚÂÛÎÄÍê³ÉÖ®¼Ê, ÎÒÏë¸ÐÐ»Ë¶Ê¿ÑÐ¾¿Éú½×¶ÎËùÓÐ°ïÖúºÍÖ§³ÖÎÒµÄÀÏÊ¦¡¢Í¬Ñ§ºÍÇ×ÓÑÃÇ.

Ê×ÏÈ, ÎÒÒª¸ÐÐ»ÎÒµÄµ¼Ê¦ÍõÎ°¸ÕÀÏÊ¦. ÎÞÂÛÔÚ¿ÆÑÐ»¹ÊÇ¹¤×÷Éú»îÖÐ, ÍõÀÏÊ¦¶¼¸øÁËÎÒºÜ´óµÄÖ¸µ¼ºÍ¹ÄÀø. ¶ÁÑÐÒÔÀ´, ÎÒÇ°½øµÄÃ¿Ò»²½±³ºó¶¼Çã×¢ÁËÍõÀÏÊ¦´óÁ¿µÄÐÄÑª. ÍõÀÏÊ¦Ñ§Ê¶Ô¨²©¡¢¹¦µ×Éîºñ, ¶ÔÓÚ¿ÆÑ§ÎÊÌâ×ÜÓÐ¶Àµ½µÄ¼û½â, ±¾ÎÄµÄ²»ÉÙ´´ÐÂµãÕýÊÇÔÚÓëÀÏÊ¦µÄÌÖÂÛµ±ÖÐµ®ÉúµÄ. ¾¡¹ÜÍõÀÏÊ¦Æ½ÈÕÀïÒª¸ºÔðÑ§ÔºÀïµÄ¹¤×÷ºÍ·±ÖØµÄ½ÌÑ§ÈÎÎñ, µ«Ëû¶ÔÓÚ¿ÆÑÐµÄÈÏÕæÌ¬¶È´Ó²»Ð¸µ¡, ¶ÔÓÚÑ§ÉúµÄÑ§Ï°ºÍÉú»îÒ²Ê±³£·ÅÔÚÐÄÉÏ. ·Ç³£ÐÒÔËÄÜÔÚË¶Ê¿ÑÐ¾¿Éú½×¶Î¸ú×ÅÍõÀÏÊ¦Ñ§Ï°, Ê¦¶÷ÄÑÍü, ÔÚ´ËÏòÍõÀÏÊ¦±íÊ¾×î³ÏÖ¿µÄ¸ÐÐ»!

Æä´Î, ÎÒÒª¸ÐÐ»ÌÖÂÛ°àµÄÃ÷ÈðÐÇÀÏÊ¦¡¢¶­Ñ©Ã·ÀÏÊ¦. Ã÷ÀÏÊ¦µÄ¿ÆÑÐÌ¬¶ÈÑÏ½÷ÈÏÕæ, ÄÜ¹»È«ÃæÉî¿ÌµØ¿´´ýÎÊÌâ, ÔÚ¿ÎÉÏ¡¢ÌÖÂÛ°àÉÏ×ÜÄÜÌá³ö´´ÐÂ¡¢×¨ÒµµÄ¹Ûµã, Ã÷ÀÏÊ¦¶ÔÓÚ±¾ÂÛÎÄµÄÑ¡ÌâºÍ´´ÐÂµãÒ²Ìá¹©ÁËºÜ´óµÄ°ïÖú; ¶­ÀÏÊ¦ÎªÈËºÍ°ª, ¾­³£ÔÚÌÖÂÛ°à¶ÔÑ§ÉúµÄÒÉÎÊ×÷³öÍ¸³¹µÄ½âÊÍ, ¸ø³öºÜ¶àÏàÓ¦µÄ½â¾ö·½°¸. Î´À´µÄÑ§Ï°ºÍ¹¤×÷ÖÐ, ÎÒ½«ÒÔÀÏÊ¦ÃÇÎª°ñÑù²»¶ÏÇ°½ø.

´ËÍâ, ÎÒÒª¸ÐÐ»ÔÚÌÖÂÛ°àÒ»ÆðÑ§Ï°µÄÐí¹¢öÎÑ§³¤ÒÔ¼°¸÷Î»Í¬Ñ§. ÐíÑ§³¤¶ÔÎÒÂÛÎÄÐ´×÷¹ý³ÌÖÐµÄÖ¸µ¼, ÈÃÎÒ»ñÒæ·ËÇ³. Ò²Òª¸ÐÐ»°àÀïµÄÍ¬Ñ§,ÔÚÆ½Ê±Ò»ÆðÑ§Ï°¡¢´òÇò¡¢½¡Éí, Ê¹ÎÒµÄÉú»î³äÂúÁËÀÖÈ¤.

×îºó, ¸ÐÐ»ÎÒ¼ÒÈË¡¢ÅóÓÑ¡¢Ç×ÆÝ, ÔÚÎÒ¶ÁÑÐÆÚ¼ä¸øÎÒµÄÉú»îÌá¹©ÁË¸÷·½ÃæµÄ¹Ø»³, Ê¹ÎÒ±£³ÖÁ¼ºÃµÄ×´Ì¬Í¶Èëµ½Ñ§ÒµÖÐ.

ÔÙ´Î¸ÐÐ»ËùÓÐ°ïÖúÎÒµÄÈËÃÇ!